

CM IMPACT Guidebook for Teachers
(With Important Questions and Answers)

Mathematics
Class-X
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Section-A
Multiples Choice Questions (MCQ) - 1 Mark

[Real Numbers]

1. For some integer p , every even integer is of the form:

- (A) p (B) $p + 1$ (C) $2p$ (D) $2p - 1$

Ans.(C)

2. The product of a non-zero rational number and an irrational number is:

- (A) An irrational number (B) a rational number (C) one (D) zero

Ans. (A)

3. The sum or difference of a rational number and an irrational number is:

- (a) A rational number (B) An irrational number (C) One (D) Zero

Ans. (B)

4. For any two positive integers p and q , $HCF(p, q) \times LCM(p, q)$ is equal to:

- (A) $p + q$ (B) $p - q$ (C) $p \times q$ (D) p/q

Ans. (C)

5. Every composite number can be expressed as a product of:

- (A) Co-primes (B) primes (C) twin primes (D) none of these

Ans. (B)

6. If p^2 is an even integer then p is a/an:

- (A) Odd integer (B) even integer (C) multiple of 3 (D) none of these.

Ans. (B)

7. The decimal expansion of a rational number is always:

- (A) Non-terminating (B) non-terminating and non- repeating

- (C) terminating or non- terminating repeated (D) none of these

Ans. (C)

8. The HCF of p and q which are relatively primes is:

- (A) 1 (B) p (C) q (D) pxq

Ans. (A)

9. The LCM of p and q which are relatively primes is:

- (A) 1 (B) p (C) q (D) $p q$

Ans. (D)

10. The product of prime factors of 156 is:

- (A) $2 \times 3 \times 13$ (B) $2^2 \times 3 \times 13$ (C) $2 \times 3^2 \times 13$ (D) $2^2 \times 3^2 \times 13$

Ans. (B)

11. Prime factors of 4050 is:

- (A) $2 \times 3^2 \times 5$ (B) $2 \times 3^4 \times 5$ (C) $2 \times 3^4 \times 5^2$ (D) $2 \times 3^4 \times 5^3$

Ans. (C)

12. If $\text{HCF}(306, 657) = 9$, then the $\text{LCM}(306, 657)$ is:

- (A) 2236 (B) 2338 (C) 22338 (D) 757

Ans. (C)

13. A number which cannot be expressed in the form a/b , where 'a' and 'b' are both integers and $b \neq 0$ is called a/an:

- (A) Rational number (B) irrational number (C) composite number (D) prime number

Ans. (B)

14. A number which can be expressed in the form a/b , where 'a' and 'b' are both integers and $b \neq 0$ is called a/an:

- (A) Rational number (B) irrational number (C) composite number (D) prime number

Ans. (A)

15. A number which is not divisible by 2 is called a/an:

- (A) Even natural number (B) whole number (C) odd natural number (D) prime number

Ans. (C)

16. A natural number which has exactly two factors i. e., 1 and the number itself is called a:

- (A) Rational number (B) whole number (C) composite number (D) prime number
- Ans. (D)

17. A natural number which is not prime and has more than two factors is called a/an:

- (A) composite number (B) whole number (C) odd natural number (D) prime number

Ans. (A)

18. A Prime number has exactly:

- (A) One factor (B) two factors (C) three factors (D) more factors

Ans. (B)

[Polynomials]

19. Which of the following is a quadratic polynomial?

- (A) $x + 7$ (B) $x^2 - 2$ (C) $x^3 + 4x + 9$ (D) $x^4 + 3x^3 + 2x + 7$

Ans. (B)

20. A polynomial of degree 3 is called a:

- (A) Linear polynomial (B) quadratic polynomial
(C) cubic polynomial (D) biquadratic polynomial

Ans. (B)

21. A quadratic polynomial can have at most:

- (A) 1 zero (B) 2 zeroes (C) 3 zeroes (D) 4 zeroes

Ans. (B)

22. The degree of a constant polynomial is:

- (A) 2 (B) 1 (C) -1 (D) 0

Ans. (D)

23. The degree of a zero polynomial is:

- (A) Always zero (B) never zero (C) negative (D) undefined

Ans. (D)

24. The degree of the polynomial $p(x) = x^2 - 5x + 6$ is:

- (A) 1 (B) 2 (C) 3 (D) 4

Ans. (B)

25. Sum of zeroes of the polynomial $p(x) = x^2 - 3x + 2$ is:

- (A) 2 (B) 3 (C) -2 (D) -3

Ans. (B)

Hint: sum of zeroes of $p(x) = -\frac{\text{co-efficient of } x}{\text{co-efficient of } x^2}$

26. Product of zeroes of the polynomial $p(x) = x^2 - 3$ is:

- (A) -3 (B) 3 (C) $\sqrt{3}$ (D) $-\sqrt{3}$

Ans. (A)

Hint: product of zeroes of $p(x) = \frac{\text{constant term}}{\text{co-efficient of } x^2}$

27. Number of zeroes of a polynomial of degree n is:

- (A) Equal to n (B) greater than n (C) less than n (D) less than or equal to n

Ans. (D) less than or equal to n

28. If the graph of $y = p(x)$, where $p(x)$ is a polynomial, does not intersect the x -axis then the number of zeros is:

- (A) 1 (B) 2 (C) 3 (D) No Zeros

Ans. (D)

29. If the graph of $y = p(x)$ where $p(x)$ is a polynomial, intersects the x -axis at one point only then the number of zeros is

- (A) 1 (B) 2 (C) 3 (D) 4

Ans. (A)

30. At most how many zeroes a linear polynomial can have?

- (A) 0 (B) 1 (C) 2 (D) 3

Ans.(B) 1

31. The zero of a linear polynomial $P(x) = ax + b$, where a, b are real numbers, is:

- (A) $-a/b$ (B) $-b/a$ (C) $-(ab)$ (D) a/b

Ans. (B) $-b/a$

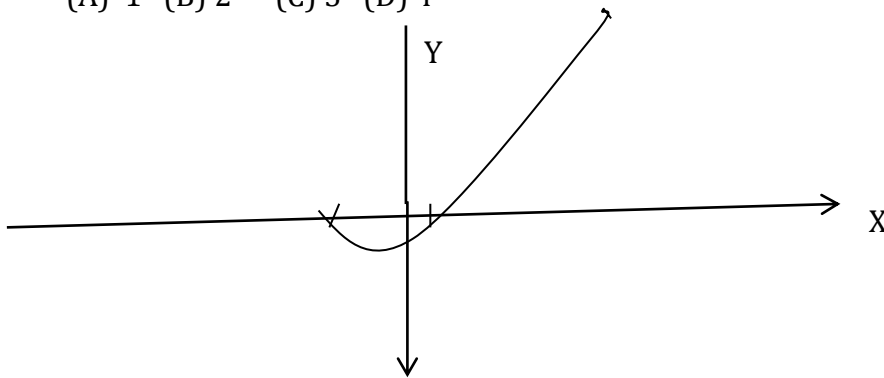
32. For any polynomial $p(x)$, if $p(a) = 0$, then 'a' is called:

- (A) Constant of the polynomial (B) zero of the polynomial
(C) Degree of the polynomial (D) coefficient of the polynomial

Ans.(B) zero of the polynomial

33. From the graph $y = p(x)$ given below, for some polynomial $p(x)$, the number of zeroes is:

- (A) 1 (B) 2 (C) 3 (D) 4



Ans. B

34. A bi-quadratic polynomial is a polynomial of degree:

- (A) 1 (B) 2 (C) 3 (D) 4

Ans. (D) 4

35. The sum and product of the zeroes of a quadratic polynomial $ax^2 + bx + c$ are:

- (A) $-b/a, c/a$ (B) $b/a, -c/a$ (C) $b/a, c/a$ (D) $-b/a, -c/a$

Ans. (A)

36. The sum and product of the zeroes of a quadratic polynomial $k^2x^2 - kx + 1$ are:

- (A) $1/k, -1/k$ (B) $-1/k, 1/k^2$ (C) $1/k, 1/k^2$ (D) $1/k, -1/k^2$

Ans. (C)

37. The zeroes of a quadratic polynomial $3x^2 - x - 4$ are:

- (A) $1, -4/3$ (B) $-1, 3/4$ (C) $-1, 4/3$ (D) $-1, -4/3$

Ans. (C)

38. The zeroes of a quadratic polynomial $x^2 - 15$ are:

- (A) $\sqrt{15}, \sqrt{15}$ (B) $\sqrt{15}, -\sqrt{15}$ (C) $3\sqrt{5}, -3\sqrt{5}$ (D) $5\sqrt{3}, -5\sqrt{3}$

Ans. (B)

39. The quadratic polynomial with 2 as sum and -8 as product of its zeroes is:

- (A) $x^2 - 2x - 8$ (B) $x^2 + 2x - 8$ (C) $x^2 - 2x + 8$ (D) $x^2 + 2x + 8$

Ans. (A)

40. The quadratic polynomial with 0 and $-1/7$ as its two zeroes is:

- (A) $7x^2 + x$ (B) $x^2 - 7x$ (C) $x^2 + 7x$ (D) $7x^2 - x$

Ans. (A)

41. If the graph of $y = x^2 - 2x - 35$ cuts the x-axis at (7, 0) and (-5, 0) then the zeroes of the polynomial $x^2 - 2x - 35$ are:

- (A) 7, -5 (B) 0, -5 (C) 0, 7 (D) 0, 0

Ans. (A)

[Pair of Linear Equations in two variables]

42. If the graphs of two lines pass through the same points, then the system of equations representing these lines is:

- (A) consistent (B) inconsistent (C) consistent dependent (D) inconsistent and dependent

Ans. (C) consistent dependent

43. The pair of equations $x = 0$ and $y = 0$ has:

- (A) no solution (B) one solution (C) two solutions (D) infinitely many solutions

Ans. (B)

44. The system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has an infinite number of

Solutions if:

- (A) $a_1/a_2 \neq b_1/b_2$ (B) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ (C) $a_1/a_2 = b_1/b_2 = c_1/c_2$ (D) none of these

Ans. (C)

45. The system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has a unique solution if:

- (A) $a_1/a_2 \neq b_1/b_2$ (B) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ (C) $a_1/a_2 = b_1/b_2 = c_1/c_2$ (D) none of these

Ans. (A)

46. The system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has no solution if:

- (A) $a_1/a_2 \neq b_1/b_2$ (B) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ (C) $a_1/a_2 = b_1/b_2 = c_1/c_2$ (D) none of these

Ans. (B)

47. The pair of equations $x = a$ and $y = b$ graphically represents lines which are:

- (A) parallel (B) coincident (C) intersecting at (a, b) (D) intersecting at (b, a)

Ans. (C)

48. The system of equations $-3x + 4y = 5$ and $\frac{9}{2}x - 6y + \frac{15}{2} = 0$ has:

- (A) Unique solution (B) infinite solutions (C) no solutions (D) none of these

Ans. (B)

49. If $x = a$ and $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b are respectively:

- (A) 3 and 5 (B) 3 and 1 (C) 5 and 3 (D) -1 and -3

Ans. (B) 3 and 1

50. If $(6, k)$ is a solution of the equation $3x + y - 22 = 0$ then the value of k is:

- (A) -4 (B) -3 (C) 4 (D) 3

Ans. (C)

[Quadratic Equations]

51. Which of the following is a quadratic equation?

- (A) $x^2 - 2x = (-2)(3 - x)$ (B) $x^2 + 3\sqrt{x} + 2 = 5$ (C) $(x - 2)(x + 1) = (x - 1)(x + 3)$
(D) $(x + 2)^3 = 2x(x^2 - 1)$

Ans. (A)

52. If the roots of the equation $ax^2 + bx + c = 0$ are equal, then c equals to:

- (A) $\frac{b}{2a}$ (B) $-\frac{b}{2a}$ (C) $\frac{b^2}{4a}$ (D) $-\frac{b^2}{4a}$

Ans. (C)

53. If $x = 3$ is a solution of the quadratic equation $3x^2 + (k - 1)x + 9 = 0$, then k equals to:

- (A) 11 (B) -11 (C) 13 (D) -13

Ans.(B)

54. The roots of the equation $ax^2 + bx + c = 0$ are non-real if:

- (A) $b^2 - 4ac = 0$ (B) $b^2 - 4ac > 0$ (C) $b^2 - 4ac < 0$ (D) $b = 0$

Ans. (C)

55. If $x = 1$ is the common root of $ax^2 + bx + 2 = 0$ and $x^2 + x + b = 0$, then a equals:

- (A) 1 (B) 0 (C) 3 (D) 4

Ans. (B)

56. A quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ will have equal roots if:

- (A) $b^2 - 4ac = 0$ (B) $b^2 - 4ac > 0$ (C) $b^2 - 4ac < 0$ (D) $b = 0$

Ans. (A)

57. The roots of the quadratic equation $100x^2 - 20x + 1 = 0$ are:

- (A) 10, 10 (B) -10, 10 (C) $1/10, 1/10$ (D) $1/10, -1/10$

Ans. (C)

58. If $\frac{1}{2}$ is a root of the quadratic equation $x^2 - kx + 6 = 0$ then the value of k is:

- (A) 25 (B) -25 (C) $25/2$ (D) $25/4$

Ans. (C)

59. If the roots of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ are real and equal, then which of the following relation is true?

- (A) $a = b^2/c$ (B) $b^2 = ac$ (C) $ac = b^2/4$ (D) $c = b^2/a$

Ans. (C)

60. If 5 pencils and 7 pen together cost ₹ 50, whereas 7 pencils and 5 pens together costs ₹ 46, then the cost of one pen is:

- (A) ₹ 3 (B) ₹ 7 (C) ₹ 5 (D) ₹ 12

Ans. (C) ₹ 5

61. If 10 students of class X took part in a Mathematics challenge during the Talent Fest organized by the School, and if the number of girls is 4 more than the number of boys, then the number of boys is:

- (A) 3 (B) 4 (C) 5 (D) 6 Ans. (A)

62. The Discriminant of the quadratic equation $x^2 + 8x + 16 = 0$ is:

- (A) 3 (B) 2 (C) 1 (D) 0 Ans. (D)

63. The solution of $(x - 4)(x + 2) = 0$ is:

- (A) 4, -2 (B) -4, 2 (C) 2, -2 (D) 4, 2 Ans. (A)

64. If $b^2 - 4ac > 0$, then the quadratic equation $ax^2 + bx + c = 0$ has:

- (A) real and equal roots (B) real and unequal roots
(C) no real roots (D) none of the above

Ans. (B)

65. If $b^2 - 4ac = 0$, then the roots of the quadratic equation $ax^2 + bx + c = 0$ are:

- (A) $-b/2a, -b/2a$ (B) $-b/2a, b/2a$ (C) $2b/a, -2b/a$ (D) $a/2b, -a/2b$

Ans. (A)

66. . If $2x^2 - 2kx + 1 = 0$ has real and equal roots, then the value of k is:

- (A) 1 (B) 2 (C) ± 2 (D) 4

Ans. (C)

67. The quadratic formula of the quadratic equation $ax^2 + bx + c$ where a, b, c are real numbers and $a \neq 0$ is given by x equals to:

- (A) $\frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$ (B) $\frac{-b \pm \sqrt{b^2 + 4ac}}{a}$ (C) $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (D) $\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

Ans. (C)

68. If the value of $(b^2 - 4ac)$ is negative for the quadratic equation $ax^2 + bx + c$ where a, b, c are real numbers and $a \neq 0$, then the nature of its roots are:

- (A) not real (B) real and unequal (C) real and equal (D) none of these

Ans. (A) not real

69. The nature of roots of the quadratic equation: $49x^2 + 21x + 9/4$ are:

(A) not real (B) real and unequal (C) real and equal (D) none of these

Ans. (C) real and equal

[Arithmetic Progression]

70. In an AP, if $d = 2$ and fourth term, $a_4 = 13$ then a is:

(A) 6 (B) 7 (C) 20 (D) 28

Ans. (B) 7

71. In an AP, if $a = 2$, $d = 0$, $n = 1000$ then a_n , is :

(A) 2 (B) 10 (C) 1000 (D) 2000

Ans. (A)

72. In an AP: 3, 1, -1, -3, the first term 'a' and common difference 'd' are respectively:

(A) 3, 2 (B) 1, -2 (C) 2, 3 (D) 3, -2

Ans. (D)

73. Which of the following series form an AP?

(A) 2, 4, 8, 16, (B) 0, -4, -8, -12,

(C) 1, 3, 9, 27, (D) 0.2, 0.22, 0.222, 0.2222,

Ans. (B)

74. The first three terms of an AP when the first term, $a = 10$ and common difference, $d = 10$ are:

(A) 10, 20, 30 (B) 0, 10, 100 (C) 10, -10, -20 (D) 10, 5, 0

Ans. (A)

75. If common difference of an AP is 5, then $a_{16} - a_{15}$ is:

(A) 1 (B) 31 (C) 5 (D) 15

Ans. (C)

76. Sum of first 20 natural numbers is:

(A) 210 (B) 120 (C) 55 (D) 15

Ans. (A)

77. The missing term in the box of the AP: 2, □, 26, Is:

(A) 6 (B) 12 (C) 13 (D) 14

Ans. (D)

78. The missing terms in the box of the AP: □, 13, □, 3, Is:

(A) 18, 8 (B) 14, 16 (C) 16, 10 (D) 18, 10

Ans. (A)

79. If k , $2k - 1$, $k + 4$ are three consecutive terms of an AP, then the value of k is:

(A) -2 (B) 3 (C) -3 (D) 6

Ans. (B)

80. If $5n + 3$ is the n th term of an AP, then the common difference is:

(A) 15 (B) 12 (C) 5 (D) 1

Ans. (C)

81. The first three terms of an AP if $a_n = 2n + 5$ are:

(A) 1, 2, 3 (B) 7, 10, 13 (C) 7, 9, 11 (D) 11, 13, 17 Ans. (C)

82. The sum of first n natural numbers is:

(A) $n(n+1)/2$ (B) $n(n+1)$ (C) $n^2 + 2n/2$ (D) n^3 Ans. (A)

83. The sum of first n terms of an AP, whose first term 'a' and last term 'l' is:

(A) $n/2 (a + 2l)$ (B) $n/2 (a + l)$ (C) $(a + l)$ (D) $n/2$ Ans. (B)

84. Wansuk saves some money from her pocket money, first month she saves ₹ 20; she decided to ₹ 10 more from second month onwards. She will be able to save ₹ 100 in:

(A) 6 months (B) 8 months (C) 9 months (D) 10 months Ans. (C)

85. In an A.P: $a, a + d, a + 2d, a + 3d, \dots$, its general term t_n equals to:

(A) $a + n d$ (B) $\{a + (n - 1)d\}$ (C) $(a + n)/2$ (D) $\{n (a + n)d\}$ Ans. (B)

86. If 'a' is the first term 'd' the common difference of an A P, then sum of first n terms is:

(A) $n/2 \{(a + (n - 1) d)\}$ (B) $n/2 \{(2a + n) d\}$
(C) $n/2 \{2a + (n - 1) d\}$ (D) $n \{ 2a + (n - 1) d\}$ Ans. (C)

87. In an AP, the difference between t_{n+1} and t_n is called the:

(A) First term (B) last term (C) common difference (D) next term

Ans. (C)

88. If $a = - 2, d = 5$ then the value of t_{10} is equal to:

(A) 23 (B) 33 (C) 43 (D) 53

Ans. (C)

89. The common difference of the AP: $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \dots$ is:

(A) $1/4$ (B) $1/2$ (C) $3/4$ (D) 0 Ans. (A)

[Triangles]

90. All geometrical congruent figures are:

(A) Not similar (B) similar (C) unequal (D) none of the above Ans. (B)

91. The ratio of any two corresponding sides in two equiangular triangles is always:

(A) The same (B) different (C) similar (D) none of the above

Ans. (A)

92. If two angles of one triangle are respectively equal to two angles of another triangles then the two triangles are similar. This is referred to as the:

(A) AA Similarity Criterion for two triangles (B) SAS Similarity Criterion for two triangles
(C) AAA Similarity Criterion for two triangles (D) SSS Similarity Criterion for two triangles

Ans. (A)

93. If $\triangle ABC$ and $\triangle DEF$ are two triangles in which $\frac{AB}{DE} = \frac{BC}{DF}$ then the two triangles are similar if:

(A) $\angle A = \angle F$ (B) $\angle B = \angle D$ (C) $\angle A = \angle D$ (D) $\angle B = \angle E$ Ans. (B)

94. The area of an equilateral triangle of side 'a' is:

(A) $3a^2$ (B) $2\sqrt{3}/a$ (C) $\sqrt{3}/4 a$ (D) $\sqrt{3}/4 a^2$ Ans. (D)

95. The length of the altitude of an equilateral triangle of side 2 cm is:

(A) 3 (B) $\sqrt{3}$ (C) $\sqrt{3}/2$ (D) $2\sqrt{3}$ Ans. (B)

96. If $\triangle ABC \sim \triangle DEF$ and $\angle A = 47^\circ$ and $\angle E = 83^\circ$, then $\angle C$ is:

(A) 50° (B) 47° (C) 80° (D) 83° Ans. (A)

97. In a triangle, if the perpendicular from the vertex to the base bisects the base. The triangle is:

(A) Scalene (B) isosceles (C) obtuse -angled (D) right- angled Ans. (B)

98. If the bisector of an angle of a triangle bisects the opposite side, then the triangle is:

(A) Scalene (B) isosceles (C) equilateral (D) right- angled Ans. (C)

99. In a triangle, the line segment joining from one vertex to the mid -point of the opposite side is called its:

(A) Median (B) perpendicular (C) hypotenuse (D) angle bisector Ans. (A)

[Coordinate Geometry]

100. The perimeter of a circle is called its:

(A) Area (B) diameter (C) radius (D) circumference

Ans. (D) circumference

101. The coordinates of a point on Y-axis is of the form:

(A) (x, y) (B) (x, 0) (C) (y, 0) (D) (0, y) Ans. (D)

102. The distance of any point P (x, y) from Origin is:

(A) $\sqrt{x^2 - y^2}$ (B) $\sqrt{y^2 - x^2}$ (C) $\sqrt{x^2 + y^2}$ (D) $\sqrt{x^3 + y^3}$

Ans. (C)

103. In which quadrant does the point (-3, 5) lie?

(A) first quadrant (B) second quadrant (C) third quadrant (D) fourth quadrant

Ans. (B)

104. The distance of the point P (3, 4) from the origin is:

(A) 4 unit (B) 3 units (C) 5 units (D) 7 units

Ans. (C)

105. The coordinates of the midpoint of the line segment joining the points A(- 2, 8) and B(- 6, - 4) is:

- (A) (4, 2) (B) (- 4, 2) (C) (4, - 2) (D) (- 4, - 2) Ans. (B)

106. The coordinates of the midpoint of the line segment joining the points A(a, b) and B(0, 0) is:

- (A) (a + b/2, a) (B) (a + b, b) (C) (a/2, b/2) (D) (a, b) Ans. (C)

107. The coordinates of a point on X –axis are of the form:

- (A) (0, x) (B) (x, 0) (C) (0, y) (D) (y, 0) Ans. (B)

108. If A (x₁, y₁), B (x₂, y₂) and C (x₃, y₃) are the three vertices of a triangle then area of a ΔABC is:

- (A) $\frac{1}{2} |x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)|$
(B) $\frac{1}{2} |x_2 (y_3 - y_1) + x_3 (y_1 - y_2) + x_3 (y_1 - y_2)|$
(C) $\frac{1}{2} |x_3 (y_2 - y_1) + x_2 (y_3 - y_1) + x_1 (y_3 - y_2)|$
(D) $\frac{1}{2} |x_1 (y_2 - y_1) + x_2 (y_3 - y_2) + x_3 (y_1 - y_3)|$

Ans. (A)

109. Three points in a plane are collinear if the area of a triangle is:

- (A) Sum of distances (B) Difference of distances (C) 1 (D) 0

Ans. (D)

110. Which one of the following is the Hero's Formula of finding the area of a triangle?

- (A) $\frac{1}{2}$ base X height (B) $\frac{1}{2} |x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)|$
(C) $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, c are the sides of a triangle and $s = \frac{a+b+c}{2}$
(D) None of the above

Ans. (C)

111. The point (3, - 4) lies in the:

- (A) First quadrant (B) second quadrant (C) third quadrant (D) fourth quadrant

Ans. (D)

112. The coordinates of the point P (x, y) which divides the line segment joining the points A(x₁, y₁) and B(x₂, y₂) in the ratio m:n is:

- (A) $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$ (B) $\left(\frac{mx_2-nx_1}{m+n}, \frac{my_2-ny_1}{m+n}\right)$
(C) $\left(\frac{mx_2+nx_1}{m-n}, \frac{my_2+ny_1}{m-n}\right)$ (D) $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

Ans. (C)

113. The coordinates of centroid of a triangle with vertices (x₁, y₁), (x₂, y₂) and (x₃, y₃) are:

- (A) $\left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}\right)$ (B) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$
(C) $\left(\frac{x_1+x_2+x_3}{6}, \frac{y_1+y_2+y_3}{6}\right)$ (D) $\left(\frac{x_1+x_2-x_3}{3}, \frac{y_1+y_2-y_3}{3}\right)$

Ans. (B)

114. The coordinates of a mid- point of the line segment AB with end points A (x_1, y_1), B (x_2, y_2) is:

- (A) $\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right)$ (B) $\left(\frac{x_1+x_2}{3}, \frac{y_1+y_2}{3}\right)$
(C) $\left(\frac{x_1+x_2}{4}, \frac{y_1+y_2}{4}\right)$ (D) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Ans. (D)

115. The perpendicular distance of a point from Y- axis called:

- (A) Ordinate (B) abscissa (C) altitude (D) none of the above Ans. (B)

116. The ordinates of all points on a horizontal line are:

- (A) Parallel (B) perpendicular (C) equal (D) coincident Ans. (C)

117. The abscissa of any point on the Y – axis is:

- (A) 1 (B) - 1 (C) 0 (D) 2 Ans. (C)

118. If P ($a/3, 4$) is the mid- point of the line segment joining the points A (- 6, 5) and B (- 2, 3) then 'a' equals:

- (A) - 6 (B) - 4 (C) - 12 (D) 12 Ans. (C)

119. The distance between the points A (4, k) and B (1, 0) is 5 units then k equals:

- (A) 4 (B) - 4 (C) 0 (D) ± 4 Ans. (D)

120. The distance between the points P (0, 5) and Q (- 5, 0) is:

- (A) 5 units (B) $5\sqrt{2}$ units (C) $2\sqrt{5}$ units (D) $\sqrt{10}$ units Ans. (B)

121. If the end points of a diameter of a circle are (1, 2) and (3, 4) then the coordinates of the Centre are:

- (A) (2, 4) (B) (2, 3) (C) (1, 2) (D) (4, 6) Ans. (B)

123. The coordinates of reflection of the point P (- 1, - 3) in X –axis are:

- (A) (- 1, 3) (B) (1, 3) (C) (1, - 3) (D) none of the above Ans. (A)

124. Centroid of a triangle is the point of concurrency of its three:

- (A) Angle bisectors (B) medians (C) altitudes (D) perpendicular bisectors

Ans. (B)

125. If 10 is the length of the line segment joining the origin from the point P (x, 8), then x is:

- (A) 6 (B) 7 (C) 9 (D) 12 Ans. (A)

126. If A (- 1, 0), B (5, - 2) and C (8, 2) are the vertices of a triangle ABC, then its centroid is:

- (A) (6, 0) (B) (0, 6) (C) (4, 0) (D) (12, 0)

Ans. (C)

[Introduction to Trigonometry]

127. The value of $1 + \tan^2 45^\circ$ is:

- (A) - 1 (B) 0 (C) 1 (D) 2

Ans. (D)

128. If $\cos\theta = 1$, then the value of θ is:

- (A) 0° (B) 30° (C) 60° (D) 90°

Ans. (A)

129. The value of $3 \cot^2 A - 3 \operatorname{Cosec}^2 A$ is equal to:

- (A) - 3 (B) 0 (C) 3 (D) $3/2$

Ans. (A)

130. In ΔABC right angle at B, if $AC = 13\text{cm}$, $BC = 5\text{ cm}$ and $AB = 12\text{ cm}$ then $\sin A$ is equal to:

- (A) $13/5$ (B) $5/13$ (C) $12/13$ (D) $13/12$

Ans. (B)

131. The value of $9 \sec^2\theta - 9 \tan^2\theta$ is:

- (A) 0 (B) 1 (C) 9 (D) 10

Ans. (C)

132. The value of $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$ is equal to:

- (A) $\cos 60^\circ$ (B) $\sin 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

Ans. (C)

133. $\sin 2A = 2 \sin A$ is true when A equals to:

- (A) 0° (B) 30° (C) 45° (D) 60°

Ans. (A)

134. If $\cos(\alpha + \beta) = 0$, then the value of $\cos((\alpha + \beta)/2)$ is equal to:

- (A) $1/\sqrt{2}$ (B) $1/2$ (C) 0 (D) $\sqrt{2}$

Ans. (A)

135. The value of $\sec 60^\circ$ is:

- (A) $\sqrt{3}/2$ (B) $1/2$ (C) 2 (D) 1

Ans. (C)

136. The relation between $\sin \theta$, $\cos \theta$ and $\tan \theta$ is: 1

- (A) $\cos \theta / \sin \theta = \tan \theta$ (B) $\sin \theta / \cos \theta = \tan \theta$
(C) $\tan \theta / \sin \theta = \cos \theta$ (D) $\tan \theta / \cos \theta = \sin \theta$

Ans. (B)

137. The relation between $\sin \theta$, $\cos \theta$ and $\cot \theta$ is: 1

- (A) $\cos \theta / \sin \theta = \cot \theta$ (B) $\sin \theta / \cos \theta = \tan \theta$
(C) $\tan \theta / \sin \theta = \cos \theta$ (D) $\tan \theta / \cos \theta = \sin \theta$

Ans. (A)

138. If $\theta = 30^\circ$ then the value of $\cos^2\theta - \sin^2\theta$ is:

- (A) 1 (B) 1/2 (C) -1/2 (D) -1

Ans. (B)

139. The reciprocal of cosine θ is:

- (A) $\tan \theta$ (B) $\sec \theta$ (C) $\operatorname{cosec} \theta$ (D) $\sin \theta$

Ans. (B)

140. $\tan^2 \theta + 1$ is equal to:

- (A) $\cot^2 \theta$ (B) $\sec^2 \theta$ (C) $\operatorname{cosec}^2 \theta$ (D) $\cos^2 \theta$

Ans. (B)

141. $1 - \sin^2 \theta$ is equal to:

- (A) $\cot^2 \theta$ (B) $\sec^2 \theta$ (C) $\operatorname{cosec}^2 \theta$ (D) $\cos^2 \theta$

Ans. (D)

[Circles]

142. If tangents PA and PB from a point P to a circle with Centre O are inclined to each other at an angle of 80° then $\angle POA$ is equal to:

- (A) 50° (B) 60° (C) 70° (D) 80° ANS. (A)

143. From a point Q, the length of the tangent to a circle is 4 cm and the distance of Q from the Centre is 5 cm, then the radius of a circle is:

- (A) 1 cm (B) 5 cm (C) 3 cm (D) 4 cm Ans. (C)

144. The tangents drawn at the end points of a diameter of a circle are:

- (A) Equal (B) parallel (C) perpendicular (D) intersecting

Ans. (B)

145. The distance between two parallel tangents of a circle of radius 8 cm is:

- (A) 8 cm (B) 12 cm (C) 14 cm (D) 16 cm

Ans. (D)

146. A part of the circle whose end points are end point of a diameter is called a:

- (A) Circumference (B) segment (C) semicircle (D) perimeter

Ans. (C)

147. The perimeter of a scalene triangle having sides 15 cm, 14 cm, 13 cm is:

- (A) 42 cm (B) 52 cm (C) 72 cm (D) 84 cm

Ans. (A)

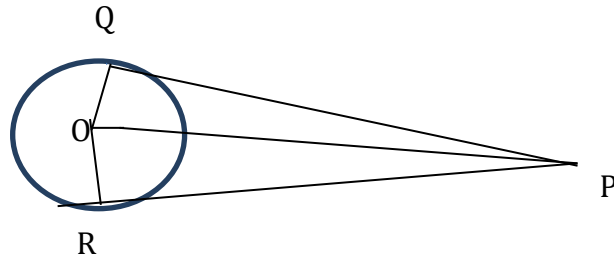
148. The perimeter of an equilateral triangle with side 9 cm is:

- (A) 9 cm (B) 18 cm (C) 27 cm (D) 36 cm

Ans. (C) 27 cm

149. In the figure, PQ and PR are tangents to a circle with Centre O and radius 5 cm. if $OP = 13$ cm, then perimeter of a quadrilateral PQOR is:

- (A) 20 cm (B) 34 cm (C) 18 cm (D) 24 cm



Ans. (B)

[Area related to Circles]

150. The portion (or part) of a circular region enclosed between a chord and the corresponding arc is called a/an:

- (A) Arc of the circle (B) perimeter of a circle (C) sector of a circle (D) segment of a circle.

Ans. (D)

151. The portion (or part) of the circular region enclosed by two radii and the corresponding arc is called the/an:

- (A) Arc of the circle (B) perimeter of a circle (C) sector of a circle (D) segment of a circle

Ans. (C)

152. If r is the radius of a circle and θ is the angle subtended by an arc of length l , then area of the sector of a circle is:

- (A) $1/2 lr$ (B) $2\pi rl$ (C) $2/3 rl$ (D) $1/2 lr\theta$

Ans. (A)

153. The Area of a circle is 49π cm². Its circumference is:

- (A) 7π cm (B) 14π cm (C) 21π cm (D) 28π cm

Ans. (B)

154. In a circle of radius 21cm, an arc subtends an angle of 60° at the centre. The length of an arc is: (take $\pi = 22/7$)

- (A) 22 cm (B) 44 cm (C) 132 cm (D) 231 cm

Ans. (A)

155. The angle made by the minute hand of a clock at its centre in 15 minutes duration is:

- (A) 60° (B) 80° (C) 90° (D) 180°

Ans. (C)

156. If the area and circumference of a circle are numerically equal, then its diameter is:

- (A) 2 units (B) 3 units (C) 4 units (D) 6 units

Ans. (C)

157. If the circumference of a circle increases from 2π to 4π , then its area is:

- (A) four times (B) tripled (C) doubled (D) halved

Ans. (A)

158. The total surface area of a hemispherical object of radius r , is:

- (A) πr^2 (B) $2\pi r^2$ (C) $3\pi r^2$ (D) $4\pi r^2$

Ans. (C)

159. The length of a diagonal of a cube of side 'a' is:

- (A) $a\sqrt{3}$ (B) $3\sqrt{a}$ (C) $\sqrt{3}a$ (D) $a/\sqrt{3}$

Ans. (C)

160. The area of square is the same as the area of circle. Their perimeters are in the ratio of:

- (A) 1:1 (B) $2:\pi$ (C) $\pi:2$ (D) $2:\sqrt{\pi}$

Ans. (D)

161. A Bicycle wheel makes 1000 revolutions in moving 88000 m. the diameter of a wheel is:

(take $\pi = 22/7$)

- (A) 14 m (B) 24 m (C) 28 m (D) 40 m

Ans. (C)

162. A garden roller has circumference of 4 m. the number of revolutions it makes in moving 40 m is:

- (A) 8 (B) 10 (C) 12 (D) 16

Ans. (B)

163. The angle described by a minute hand in 1 hour is:

- (A) 60° (B) 120° (C) 180° (D) 360°

Ans. (D)

164. The circumference of a circle of diameter D units is:

- (A) πD (B) $2\pi D$ (C) $4\pi/2$ (D) $\pi D/2$

Ans. (A) πD

165. If 'r' and 'h' represent the radius of the base and height of a right circular cone respectively then its curved surface area is:

- (A) $\pi r h$ (B) $\pi r^2 h$ (C) $\pi r (\sqrt{r^2 + h^2})$ (D) $\pi r h^2$

Ans. (C)

166. The area of a sector whose perimeter is four times its radius 'r' units is:

- (A) $r^2/4$ (B) $2r^2$ (C) r^2 (D) $r^2/2$

Ans. (C)

167. Distance covered by a wheel in one revolution is equal to:

- (A) Diameter of a wheel (B) radius of a wheel
(C) Circumference of a wheel (D) none of these

Ans. (C)

168. The circumference of a bicycle wheel that makes 5000 revolutions in moving 11 km is:

(take $\pi = 22/7$)

- (A) 500 cm (B) 250 cm (C) 220 cm (D) 150 cm

Ans. (C)

169. The area of a circle when area and circumference are numerically equal is:

- (A) 2π (B) 4π (C) 6π (D) 8π

Ans. (B)

170. When area and circumference are numerically equal then the radius of the circle is equal to:

- (A) 1 unit (B) 2 units (C) 3 units (D) 4 units

Ans. (B)

171. The difference between circumference and diameter of a circle is 90 cm, then the radius of a circle is:

- (A) 24.5 cm (B) 23 cm (C) 22.5 cm (D) 21 cm

Ans. (D)

[Surface Area and Volumes]

172. The total surface area of a hemisphere with radius 'R' is:

- (A) πR^2 (B) $2\pi R^2$ (C) $3\pi R^2$ (D) $4\pi R^2$

Ans. (C)

173. The volume of a sphere of radius R is:

- (A) $2/3 \pi R^3$ (B) $4/3\pi R^3$ (C) $3\pi R^3$ (D) $4\pi R^3$

Ans. (B)

174. During the conversion of a solid from one shape to another, the volume of the new shape will:

- (A) Increase (B) decrease (C) remain unaltered (D) depends on the shape

Ans. (C)

175. If the radius of a sphere becomes 3 times, then its volume will become:

- (A) 3 times (B) 6 times (C) 9 times (D) 27 times

Ans. (D)

176. The surface areas of two spheres are in the ratio 16:9. The ratio of their volume is:

(A) 64:27 (B) 16:9 (C) 4:3 (D) $16^3:9^3$

Ans. (A)

177. A solid metallic sphere of radius 9 cm is melted to form a solid cylinder of radius 9 cm. the height of the cylinder is:

(A) 12 cm (B) 18 cm (C) 36 cm (D) 96 cm

Ans. (A)

178. If a cone and a sphere have equal radii and equal volumes then the ratio of the diameter of the sphere to the height of the cone is:

(A) 3:4 (B) 2:3 (C) 2:1 (D) 1:2

Ans. (D)

179. The volume of a cube of an edge of 4 cm is:

(A) 16 cm^3 (B) 64 cm^3 (C) 128 cm^3 (D) 256 cm^3

Ans.(B)

180. Eight metallic sphere each of radius 2mm are melted and recast into a single sphere. Then the radius of the new sphere is:

(A) 4 mm (B) 4.5 mm (C) 5.5 mm (D) 6 mm

Ans. (A)

181. The total surface area of a cube of length 'a' units is:

(A) $3a^2$ sq. units (B) $4a^2$ sq. units (C) $6a^2$ sq. units (D) $8a^2$ sq. units

Ans. (C)

182. The total surface area of a right circular cylinder of radius 'r' and height 'h' is:

(A) $2\pi r(r+h)$ (B) $2\pi r h$ (C) $2\pi r(r-h)$ (D) $3\pi r(r+h)$

Ans. (A)

183. If the curved surface area of a cone is 308 cm^2 and its slant height is 14 cm, then radius of the base is:

(A) 14 cm (B) 7 cm (C) 21 cm (D) 28 cm

Ans. (B)

184. If the radius of a sphere is doubled then the ratio of the volumes of the first sphere to the new sphere is:

(A) 1:2 (B) 1:4 (C) 1:6 (D) 1:8

Ans. (D)

185. Let 'r' and 'h' be the radius and height respectively of a right circular cylinder, then its curved surface area is:

(A) $2\pi r(r+h)$ (B) $2\pi r h^2$ (C) $2\pi r h$ (D) $2\pi r(r-h)$

Ans. (C)

186. Let 'r' and 'h' be the radius and height respectively of a cone, then its volume is equal to:

- (A) $\pi r^2 h$ (B) πr^2 (C) $\frac{1}{3} \pi r^2 h$ (D) $\frac{2}{3} \pi r h$

Ans. (C)

187. Let R and r be the outer and inner radii of the spherical shell, then its volume is equal to:

- (A) $\frac{4}{3} \pi (R^3 - r^3)$ (B) $\frac{4}{3} \pi (R^2 - r^2)$
(C) $\frac{2}{3} \pi (R^3 - r^3)$ (D) $\frac{2}{3} \pi (R^2 - r^2)$

Ans. (D)

[Statistics]

188. The empirical relationship between mean, mode and median in asymmetrical distribution is:

- (A) Mode = 3 Median - 2 Mean (B) Mode = 3 Median + 2 Mean
(C) Mode = 2 Median - 3 Mean (D) Mode = Median - 2 Mean

Ans. (A)

189. Mode is:

- (A) Least frequent value (B) middle most value
(C) Most frequent value (D) none of the above

Ans. (C)

190. The cumulative frequency table is useful in determining the:

- (A) Mean (B) Median (C) Mode (D) All of the above

Ans. (B)

191. If the mode of 16, 15, 17, 16, 15, x, 19, 17, 14 is 15 then x is:

- (A) 15 (B) 16 (C) 17 (D) 19

Ans. (A)

192. Which of the following cannot be the probability of an event?

- (A) $\frac{2}{3}$ (B) 1.5 (C) $\frac{1}{2}$ (D) 0.7

Ans. (B)

193. The sum of the values of all the observations divided by the total number of observations is called:

- (A) Mean (B) mode (C) median (D) frequency

Ans. (A)

194. In a frequency distribution, the class having the maximum frequency is called:

- (A) Class mark (B) class size (C) median class (D) modal class

Ans. (D)

195. The wickets taken by a bowler in 10 cricket matches are as follows:

2, 6, 4, 5, 0, 2, 1, 3, 2, 3

Then the mode of the data is:

- (A) 3 (B) 2 (C) 4 (D) 5

Ans. (B)

196. For some data x_1, x_2, \dots, x_n with respective frequencies f_1, f_2, \dots, f_n , the value of $\sum f_i (x_i - x')$ is equal to, where x' is mean:

- (A) nx' (B) 1 (C) $\sum f_i$ (D) 0

Ans. (D)

197. The middlemost observation of every data arranged in order is called:

- (A) Mode (B) median (C) mean (D) deviation

Ans. (B)

198. The mean of first 10 natural numbers is:

- (A) 5.5 (B) 5 (C) 6 (D) 10

Ans. (A)

199. The difference between the minimum and maximum values of the data is called:

- (A) class limits (B) class interval (C) class size (D) range of data

Ans. (D)

200. The value, $(\text{Lower Limit} + \text{Upper Limit})/2$, is called:

- (A) Class size (B) class limits (C) class mark (D) class interval

Ans. (C)

201. In a T20 International Cricket match, a batsman played 40 balls and the runs scored are as follows:

Runs scored	No. of ball faced
0	4
1	15
2	5
3	1
4	4
6	1

The probability that a batsman will score 1 or 2 runs is:

- (A) $1/4$ (B) $1/8$ (C) $1/2$ (D) $1/10$

Ans. (C)

202. If l = lower limit of the median class, $N/2$ = half of the cumulative frequency

c = cumulative frequency of the preceding class, h = height of each class

f = frequency of the median class. Then the formula for finding median is median equals:

- (A) $l + (N/2 - c)h/f$ (B) $l - (N/2 - c)h/f$ (C) $l + (N/2 + c)h/f$ (D) $l + (N/2)h/f$

Ans. (C)

203. The median of the following data:

6, 8, 15, 16, 9, 22, 21, 25, 18 is:

- (A) 21 (B) 18 (C) 16 (D) 9

Ans. (C)

204. If the mean of 6, 8, 5, 7, 4 and x is 7, then x equals to:

- (A) 12 (B) 24 (C) 28 (D) 30

Ans. (A)

205. If the mean of x , $x + 3$, $x + 6$, $x + 9$ and $x + 12$ is 10, then x is:

- (A) 1 (B) 2 (C) 4 (D) 6

Ans. (C)

206. Which of the following is not the measure of central tendency?

- (A) Mean (B) mode (C) median (D) standard deviation

Ans. (D)

207. The formula for finding mean by direct method is:

- (A) $x' = \sum f_i x_i / \sum f_i$ (B) $x' = \sum f_i x_i / f_i$ (C) $x' = \sum x_i / \sum f_i$ (D) $x' = \sum f_i x_i$

Ans. (A)

208. The data which have two modes are called:

- (A) Unimodal (B) multimodal (C) bimodal (D) trimodal

Ans. (C) bimodal

[Probability]

209. A die is thrown once. The probability of getting a number less than 3 is:

- (A) $1/2$ (B) $1/3$ (C) $1/4$ (D) $1/6$

Ans. (B)

210. A die is thrown once. The probability of getting a prime number is:

- (A) $1/2$ (B) $1/3$ (C) $2/3$ (D) $1/6$

Ans. (A) $1/2$

211. The probability of a sure Event is:

- (A) 0 (B) $1/2$ (C) 1 (D) non-existent

Ans. (C)

212. The probability of an impossible Event is:

- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) non-existent

Ans. (A)

213. If $P(E) = 0.05$, then $P(E')$ equals:

- (A) 0.05 (B) 0.5 (C) 0.9 (D) 0.95

Ans. (D)

214. A jar contains 6 red, 5 black, and 3 green marbles of equal size. The probability that a randomly drawn marble would be green in colour is:

- (A) $\frac{5}{14}$ (B) $\frac{11}{14}$ (C) $\frac{3}{14}$ (D) $\frac{1}{3}$

Ans. (C)

215. If the probability of a player winning a game is 0.79, then the probability of his losing the same game is:

- (A) 1.79 (B) 0.31 (C) 0.21% (D) 0.21

Ans. (D) 0.21

216. A die is thrown once, then the probability of getting an even prime number is:

- (A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

Ans. (A)

217. A letter is selected at random from the letters of the words 'MATHEMATICS' then the probability of getting the letter M is:

- (A) $\frac{2}{11}$ (B) $\frac{6}{11}$ (C) $\frac{4}{11}$ (D) $\frac{5}{11}$

Ans. (A)

218. In a musical chair game, the person playing the music has been advised to stop playing the music at any time within 2 minutes after the play starts. Then the probability that the music will stop within one and half minutes after starting the game is:

- (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{1}{3}$

Ans. (C)

219. The probability that a non-leap year has 53 Sundays is:

- (A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{5}{7}$ (D) $\frac{6}{7}$

Ans. (A) $\frac{1}{7}$

220. The probability that a number selected at random from the numbers 3,4,5,6,7, 8, 9 is a multiple of 4 is:

- (A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{5}{7}$ (D) $\frac{6}{7}$

Ans. (B)

221. A jar contains 25 marble with 10 green marbles and the rest are blue marbles. If a marble is drawn at random from the jar, then the probability that the drawn marble is blue is:

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{1}{5}$

Ans. (B)

Section-B

Very Short Answer Questions (2 Marks)

1. Express 5005 as a product of its prime factors

$$\begin{array}{r|l} \text{Ans.} & 5005 \\ \hline & 5 \ 0 \ 0 \ 5 \\ \hline & 7 \ 1 \ 0 \ 0 \ 1 \\ \hline & 11 \ 1 \ 4 \ 3 \\ \hline & 13 \end{array}$$

$$\therefore 5005 = 5 \times 7 \times 11 \times 13$$

2. Find the least Number which is exactly divisible by 15, 24 and 30.

$$\begin{array}{r|l} \text{Ans} & 15 \\ \hline & 3 \ 15 \\ \hline & 5 \end{array} \quad \begin{array}{r|l} & 24 \\ \hline & 2 \ 24 \\ \hline & 2 \ 12 \\ \hline & 2 \ 6 \\ \hline & 3 \end{array} \quad \begin{array}{r|l} & 30 \\ \hline & 2 \ 30 \\ \hline & 3 \ 15 \\ \hline & 5 \end{array}$$

$$\therefore 15 = 3 \times 5$$

$$24 = 2^3 \times 3$$

$$30 = 2 \times 3 \times 5$$

$$\therefore \text{LCM} = 2^3 \times 3 \times 5 = 120$$

$$\therefore \text{The required least number} = 120$$

3. Find the LCM of the smallest two digit number and the smallest composite number.

Ans The smallest two digit number = 10

The smallest composite number = 4

$$\therefore 10 = 2 \times 5$$

$$4 = 2^2$$

$$\therefore \text{LCM} (10, 4) = 2 \times 2 \times 5$$

$$= 20$$

4. Find the HCF and LCM of 336 and 54 by applying the fundamental theorem of Arithmetic

Ans

	2	336
2	168	
2	84	
2	42	
3	21	
	7	

	2	54
3	27	
3	9	
	3	

$$\therefore 336 = 2^4 \times 3 \times 7$$

$$54 = 2 \times 3^3$$

$$\therefore \text{HCF} = 2 \times 3 = 6$$

$$\text{LCM} = \frac{\text{Product of two numbers}}{\text{HCF}}$$

$$= \frac{336 \times 54}{6}$$

$$= 3024$$

5. Given that $\text{HCF}(306, 657) = 9$, find the $\text{LCM}(306, 657)$

Ans We know,

$$\text{HFC} \times \text{LCM} = \text{Product of two numbers}$$

$$\Rightarrow 9 \times \text{LCM} = 306 \times 657$$

$$\Rightarrow \text{LCM} = \frac{306 \times 657}{9}$$

$$\Rightarrow \text{LCM} = 22338$$

6. Prove that $7\sqrt{5}$ is irrational.

Ans Let us assume, to the contrary, that $7\sqrt{5}$ is rational

$$\therefore 7\sqrt{5} = \frac{a}{b}, \text{ where } a \text{ and } b \text{ are integers and } b \neq 0$$

$$\Rightarrow 7\sqrt{5} b = a$$

$$\Rightarrow \sqrt{5} = \frac{a}{7b}$$

But $\frac{a}{7b}$ is rational and so $\sqrt{5}$ is rational

But this contradicts the fact that $\sqrt{5}$ is irrational

Hence, $7\sqrt{5}$ is irrational

Hence Proved

7. Find the HCF and LCM of 21 and 315 by prime factorisation method.

Ans

3	21	3	315
7	3	3	105
	5	5	35
	7		

$$\therefore 21 = 3 \times 7$$

$$315 = 3^2 \times 5 \times 7$$

$$\therefore \text{HCF} = 3 \times 7 = 21$$

$$\text{LCM} = 3^2 \times 5 \times 7 = 315$$

8. Check whether 6^n can end with the digit 0 for any natural number n .

Ans If 6^n ends with the digit zero, then it should be divisible by 10

So, the prime factorization of 6^n must contain the primes 2 and 5. This is not possible as the only primes in the factorisation of 6^n are 2 and 3. Thus, there is no value of n in natural numbers for which 6^n ends with the digit zero.

9. Find the HCF and LCM of 12, 15 and 21 by applying the prime factorisation method.

Ans

2	12	3	15	3	21
2	6	5		7	
	3				

$$\therefore 12 = 2^2 \times 3$$

$$15 = 3 \times 5$$

$$21 = 3 \times 7$$

$$\therefore \text{HCF} = 3$$

$$\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$$

10. In a class there are 24 girls and 20 boys. Find the minimum number of books that can be distributed equally among girls and boys.

Ans

2	24	2	20
2	12	2	10

$$2 \frac{6}{3} \quad 5$$

$$3$$

$$\therefore 24 = 2^3 \times 3$$

$$20 = 2^2 \times 5$$

$$\therefore \text{LCM} = 2^3 \times 3 \times 5 = 120$$

\therefore Minimum number of books that can be distributed equally among girls and boys = 120

11. Check whether the system of linear equations $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$ intersect at a point or not.

Ans Here,

$$a_1 = 5, b_1 = -4, c_1 = 8$$

$$a_2 = 7, b_2 = 6, c_2 = -9$$

$$\frac{a_1}{a_2} = \frac{5}{7}$$

$$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$$

$$\frac{c_1}{c_2} = \frac{-8}{9}$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

\therefore The given system of linear equations intersects at a point.

12. Find out whether the lines representing a pair of linear equations $2x - 3y - 8 = 0$, $4x - 6y - 9 = 0$ has a unique solution or not.

Ans Here,

$$a_1 = 2, b_1 = -3, c_1 = -8$$

$$a_2 = 4, b_2 = -6, c_2 = -9$$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$$

$$\text{Here, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

\therefore The given system of linear equations does not have a unique solution.

13. Determine whether the following system of linear equations is consistent or inconsistent.

$$2x - 3y = 8$$

$$4x - 6y = -5$$

Ans Here,

$$a_1 = 2, b_1 = -3, c_1 = -8$$

$$a_2 = 4, b_2 = -6, c_2 = 5$$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{-8}{5}$$

$$\text{Here, } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

\therefore The given system of linear equations is inconsistent.

14. Check whether the following system of linear equations $9x + 3y + 12 = 0$, $18x + 6y + 24 = 0$ represent co-incident lines or not.

Ans Here,

$$a_1 = 9, b_1 = 3, c_1 = 12$$

$$a_2 = 18, b_2 = 6, c_2 = 24$$

$$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$$

$$\text{Here, } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

\therefore The given system of linear equations represents co-incident lines.

15. Find the value of k for which the system of linear equations $kx + 2y = 5$, $3x + y = 1$ has a unique solution.

Ans $kx + 2y = 5 \Rightarrow kx + 2y - 5 = 0$

$$3x + y = 1 \Rightarrow 3x + y - 1 = 0$$

Here,

$$a_1 = k, b_1 = 2, c_1 = -5$$

$$a_2 = 3, b_2 = 1, c_2 = -1$$

For the system of linear equations to have a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \frac{k}{3} \neq \frac{2}{1}$$

$$\Rightarrow k \neq 6$$

16. Solve the system of linear equations $x + y = 14, x - y = 4$ by substitution method.

Ans $x + y = 14$ -----(i)

$$x - y = 4$$
 -----(ii)

$$\Rightarrow x = 4 + y$$

Substituting the value of x in eqn (i), we get

$$x + y = 14$$

$$\Rightarrow 4 + y + y = 14$$

$$\Rightarrow 4 + 2y = 14$$

$$\Rightarrow 2y = 14 - 4$$

$$\Rightarrow 2y = 10$$

$$\Rightarrow y = \frac{10}{2}$$

$$\Rightarrow y = 5$$

Putting $y = 5$ in eqn (i), we get

$$x + y = 14$$

$$\Rightarrow x + 5 = 14$$

$$\Rightarrow x = 14 - 5$$

$$\Rightarrow x = 9$$

$$\therefore x = 9 \text{ and } y = 5$$

17. Solve the system of linear equations $3x + 4y = 10$, $2x - 2y = 2$ by elimination method.

Ans $3x + 4y = 10$ -----(i) $\times 2$

$2x - 2y = 2$ -----(ii) $\times 3$

Multiplying eqn (i) by 2 and eqn (ii) by 3, we get

$$6x + 8y = 20$$

$$6x - 6y = 6$$

$$\begin{array}{r} (-) \qquad (+) \qquad (-) \\ \hline \end{array}$$

(Subtracting) $14y = 14$

$$\Rightarrow y = \frac{14^1}{14_1}$$

$$\Rightarrow y = 1$$

Putting $y = 1$ in eqn (i), we get

$$3x + 4y = 10$$

$$\Rightarrow 3x + 4 \times 1 = 10$$

$$\Rightarrow 3x + 4 = 10$$

$$\Rightarrow 3x = 10 - 4$$

$$\Rightarrow 3x = 6$$

$$\Rightarrow x = \frac{6^2}{3_1}$$

$$\Rightarrow x = 2$$

$$\therefore x = 2$$

$$y = 1$$

18. The difference between two numbers is 26 and one number is three times the other. Find them.

Ans : Let the two number be x and y ($x > y$)

\therefore According to the question,

$$x - y = 26 \text{ -----(i)}$$

$$x = 3y \text{ -----(ii)}$$

Putting $x = 3y$ in eqn (i), we get

$$x - y = 26$$

$$\Rightarrow 3y - y = 26$$

$$\Rightarrow 2y = 26$$

$$\Rightarrow y = \frac{26}{2}$$

$$\Rightarrow y = 13$$

Putting $y = 13$ in eqn (ii), we get

$$x = 3y$$

$$\Rightarrow x = 3 \times 13$$

$$\Rightarrow x = 39$$

\therefore The two numbers are 39 and 13

19. 10 students of class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Ans Let the number of boys and girls be ' x ' and ' y ' and respectively

\therefore According to the question,

$$x + y = 10 \quad \text{-----(i)}$$

$$y = 4 + x \quad \text{-----(ii)}$$

Putting $y = 4 + x$ in eqn (i), we get

$$x + y = 10$$

$$\Rightarrow x + 4 + x = 10$$

$$\Rightarrow 2x + 4 = 10$$

$$\Rightarrow 2x = 10 - 4$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow x = \frac{6}{2}$$

$$\Rightarrow x = 3$$

Putting $x = 3$ in eqn (ii), we get

$$y = 4 + x$$

$$\Rightarrow y = 4 + 3$$

$$\Rightarrow y = 7$$

$$\therefore \text{Number of boys} = x = 3$$

$$\text{Number of girls} = y = 7$$

20. From a pair of linear equations in the given problem:

The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, she buys 3 bats and 5 balls for ₹ 1750.

Ans Let the cost of each bat be ₹ x and the cost of ball be ₹ y

\therefore The cost of 7 bats is ₹ $7x$ and the cost of 6 balls is ₹ $6y$ and their total cost is ₹ 3800

$$\therefore 7x + 6y = 3800$$

Again, the cost of 3 bats is ₹ $3x$ and the cost of 5 balls is ₹ $5y$ and their cost is ₹ 1750

$$\therefore 3x + 5y = 1750$$

21. Find the discriminant and nature of roots of the quadratic equation $2x^2 - 3x - 2 = 0$.

Ans Here,

$$a = 2, b = -3, c = -2$$

$$\therefore \text{Discriminant} = b^2 - 4ac$$

$$= (-3)^2 - 4 \times 2 \times (-2)$$

$$= 9 + 16$$

$$= 25 > 0$$

Thus, the nature of roots are real and unequal

22. Solve the quadratic equation $2x^2 = 9x$.

Ans $2x^2 = 9x$

$$\Rightarrow 2x^2 - 9x = 0$$

$$\Rightarrow x(2x - 9) = 0$$

\therefore Either, or,

$$x = 0 \quad 2x - 9 = 0$$

$$\Rightarrow 2x = 9$$

$$\Rightarrow x = \frac{9}{2}$$

$$\therefore x = 0, \frac{9}{2}$$

23. Check whether $(x + 4)(x - 4) = x(x + 2) + 8$ is a quadratic equation or not.

Ans $(x + 4)(x - 4) = x(x + 2) + 8$

$$\Rightarrow (x)^2 - (4)^2 = x^2 + 2x + 8$$

$$\Rightarrow x^2 - 16 - x^2 - 2x - 8 = 0$$

$$\Rightarrow -2x - 24 = 0$$

It is not in the form $ax^2 + bx + c = 0$

\therefore The given equation is not a quadratic equation

24. Find the roots of the quadratic equation $2x^2 + x - 6 = 0$ by factorization.

Ans $2x^2 + x - 6 = 0$

$$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$$

$$\Rightarrow 2x(x + 2) - 3(x + 2) = 0$$

$$\Rightarrow (x + 2)(2x - 3) = 0$$

\therefore Either, or,

$$x + 2 = 0 \qquad 2x - 3 = 0$$

$$\Rightarrow x = -2 \qquad \Rightarrow 2x = 3$$

$$\Rightarrow x = \frac{3}{2}$$

Hence the roots are $-2, \frac{3}{2}$

25. Represent the given situations in the form of a quadratic equations.

The area of a rectangular plot is 528m^2 . The length of the plot (in metres) is one more than twice its breadth.

Ans Let the length (in metres) of the plot be x and the breadth (in metres) of the plot be y

$$\therefore \text{Area of the plot} = (xy)\text{m}^2$$

$$\therefore xy = 528 \dots\dots\dots\text{(i)}$$

$$\text{Also, } x = 1 + 2y \dots\dots\dots\text{(ii)}$$

Using (ii) in (i), we get

$$(1 + 2y)y = 528$$

$$\Rightarrow 2y^2 + y - 528 = 0, \text{ is a quadratic equation}$$

26. Find the nature of the quadratic equation $x^2 + 7x + 10 = 0$.

Ans The quadratic equation is $x^2 + 7x + 10 = 0$

Here,

$$a = 1, b = 7, c = 10$$

$$\begin{aligned}\therefore \text{Discriminant } (D) &= b^2 - 4ac \\ &= (7)^2 - 4 \times 1 \times 10 \\ &= 49 - 40 \\ &= 9\end{aligned}$$

$$\therefore D > 0$$

\therefore The given equation has two real and distinct roots

27. State any two conditions that decides the nature of roots of a general quadratic equation $ax^2 + bx + c = 0$ where a, b, c are real numbers and $a \neq 0$

Ans . Any two conditions that decides the nature of roots of general quadratic equation are:

- 1) If $D > 0$, the equation has real and unequal roots
If D is a perfect square, the equation has unequal rational roots
- 2) If $D = 0$, the equation has real and equal roots and each root is $\frac{-b}{2a}$

$$D = b^2 - 4ac$$

28. The coach of a cricket team buys 3 bats and 2 balls for ₹ 5700. Later he buys 2 bats and 3 balls for ₹ 4050. Find the cost of one bat?

Ans Let ₹ x and ₹ y be the cost of each bat and each ball respectively

\therefore By question,

$$3x + 2y = 5700 \text{ -----(i) } \times 3$$

Also, $2x + 3y = 4050 \text{ (ii) } \times 2$

Multiplying (i) by 2 and (ii) by 3 we get

$$9x + 6y = 17100$$

$$\underline{4x + 6y = 8100}$$

Subtracting, $5x = 9000$

$$\Rightarrow x = 9000/5$$

$$\Rightarrow x = 1800$$

Thus, cost of one bat = ₹ 1800

29. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages (in years) was 48.

Ans Let the age of one of two friends be x .

Then the age of the other friend = $(20 - x)$

[\because the sum of the ages of two friends is 20 years]

4 yr ago, age of one of two friends = $(x - 4)$

And age of the other friend = $(20 - x - 4) = (16 - x)$

According to the question,

$$(x - 4)(16 - x) = 48$$

$$\Rightarrow 16x - x^2 - 64 + 4x = 48$$

$$\Rightarrow x^2 - 20x + 112 = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -20, \text{ and } c = 112$$

Now, discriminant, $D = b^2 - 4ac$

$$= (-20)^2 - 4 \times 1 \times 112$$

$$= 400 - 448$$

$$= -48 < 0$$

Which implies that the real roots are not possible as this condition represents imaginary roots. So, the solution does not exist and hence given situation is not possible.

30. Find two consecutive positive numbers whose square have the sum 85.

Ans Let the two consecutive positive numbers be x and $x + 1$

\therefore According to the question,

$$x^2 + (x + 1)^2 = 85$$

$$\Rightarrow x^2 + x^2 + 2x + 1 = 85$$

$$\Rightarrow 2x^2 + 2x + 1 - 85 = 0$$

$$\Rightarrow 2x^2 + 2x - 84 = 0$$

$$\Rightarrow 2(x^2 + x - 42) = 0$$

$$\Rightarrow x^2 + x - 42 = 0$$

$$\Rightarrow x^2 + 7x - 6x - 42 = 0$$

$$\Rightarrow x(x + 7) - 6(x + 7) = 0$$

$$\Rightarrow (x + 7)(x - 6) = 0$$

\therefore Either, or,

$$x + 7 = 0$$

$$x - 6 = 0$$

$$\Rightarrow x = -7$$

$$\Rightarrow x = 6$$

(Rejected)

\therefore The two positive numbers are 6 and 7

31. In the adjoining figure, $DE \parallel BC$. Find EC where $AD = 1.5\text{cm}$, $DB = 3\text{cm}$ and $AE = 1\text{cm}$.

Ans $\because DE \parallel BC$

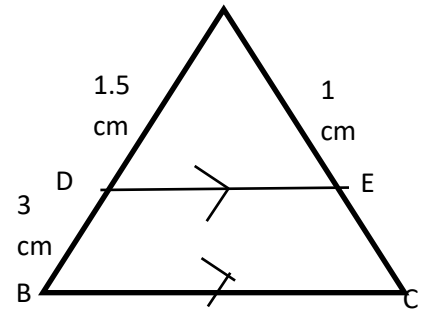
\therefore By Basic Proportionality Theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{1.5}{3} = \frac{1\text{cm}}{EC}$$

$$\Rightarrow EC = \frac{3}{1.5} = \frac{3^1 \times 10^2}{15 \cancel{5}_1}$$

$$\Rightarrow EC = 2\text{cm}$$



32. In ΔABC , D and E are points on the sides AB and AC respectively such that $DE \parallel BC$. If

$$\frac{AD}{DB} = \frac{2}{3} \text{ and } AE = 7.2\text{cm}, \text{ find } AC.$$

Ans $\because DE \parallel BC$

\therefore By Basic Proportionality Theorem,

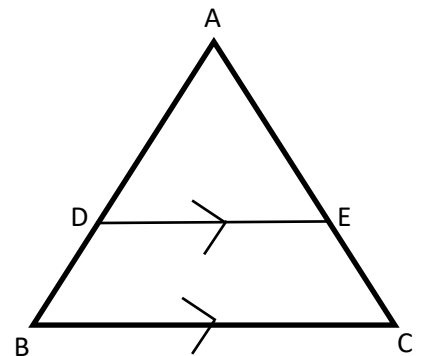
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{2}{3} = \frac{7.2\text{ cm}}{EC}$$

$$\Rightarrow EC = \frac{3 \times 7.2 \cancel{3.6}}{\cancel{2}_1} \text{ cm}$$

$$\Rightarrow EC = 10.8\text{ cm}$$

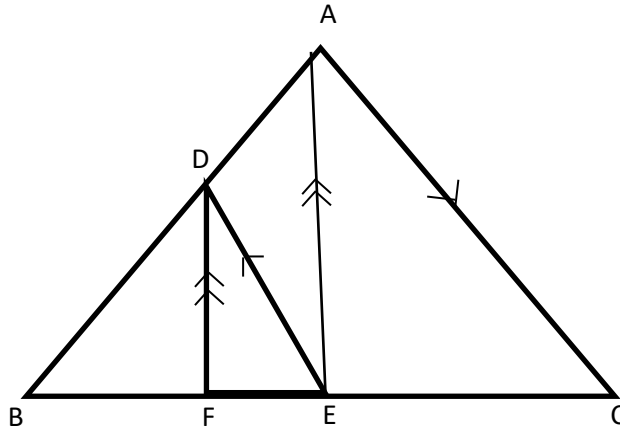
$$\therefore AC = AE + EC$$



$$= 7.2 \text{ cm} + 10.8 \text{ cm}$$

$$= 18 \text{ cm}$$

33. In the adjoining figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



Ans

In $\triangle ABC$,

$$DE \parallel AC$$

\therefore By Basic Proportionality Theorem

$$\frac{BD}{DA} = \frac{BE}{EC} \quad \text{-----(i)}$$

Again, in $\triangle ABE$,

$$DF \parallel AC$$

\therefore By Basic Proportionality Theorem

$$\frac{BD}{DA} = \frac{BF}{FE} \quad \text{-----(ii)}$$

\therefore From eqn (i) and (ii), we get

$$\frac{BF}{FE} = \frac{BE}{EC}$$

Hence Proved

34. Prove that the line drawn from the mid-point of one side of a triangle parallel to another side bisects the third side.

Ans Given:- $\triangle ABC$, in which D is the mid-point of AB and $DE \parallel BC$

To Prove:- $AE = EC$

Proof:- In $\triangle ABC$,

$\because DE \parallel BC$

\therefore By Basic Proportionality Theorem

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{-----(i)}$$

\because D is the mid-point of AB

$$\therefore AD = DB \quad \text{-----(ii)}$$

\therefore From eqn (i) and (ii), we get

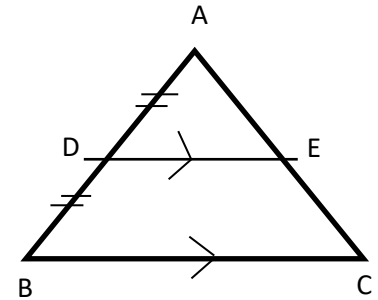
$$\frac{AD}{AD} = \frac{AE}{EC}$$

$$\Rightarrow 1 = \frac{AE}{EC}$$

$$\Rightarrow AE = EC$$

$$\Rightarrow DE \text{ bisects } AC$$

Hence Proved



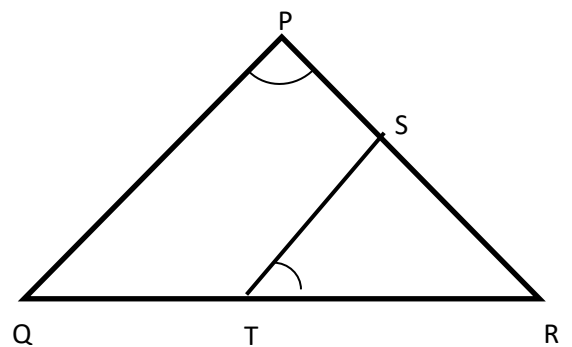
35. S and T are points on side PR and QR of $\triangle PQR$ such that $\angle P = \angle RST$. Show that $\triangle RPQ \sim \triangle RST$.

Ans In $\triangle RPQ$ and $\triangle RST$

$\angle P = \angle RST$ (Given)

$\angle PRQ = \angle SRT$ (Common angle)

$\therefore \triangle RPQ \sim \triangle RST$ [AA Similarity]



Hence Showed

36. In the adjoining figure, $\Delta OAB \sim \Delta OCD$. When $AB = 8\text{cm}$, $BO = 6.4\text{cm}$, $OC = 3.5\text{cm}$ and $CD = 5\text{cm}$, find OA and DO .

Ans

$$\because \Delta OAB \sim \Delta OCD$$

$$\therefore \frac{AB}{CD} = \frac{OA}{OC} = \frac{OB}{OD}$$

$$\Rightarrow \frac{8}{5} = \frac{OA}{3.5\text{ cm}} = \frac{6.4\text{ cm}}{OD}$$

$$\text{Taking, } \frac{8}{5} = \frac{OA}{3.5\text{ cm}}$$

$$\Rightarrow 5 \times OA = 8 \times 3.5\text{ cm}$$

$$\Rightarrow OA = \frac{8 \times 3.5^{0.7}\text{ cm}}{5_1}$$

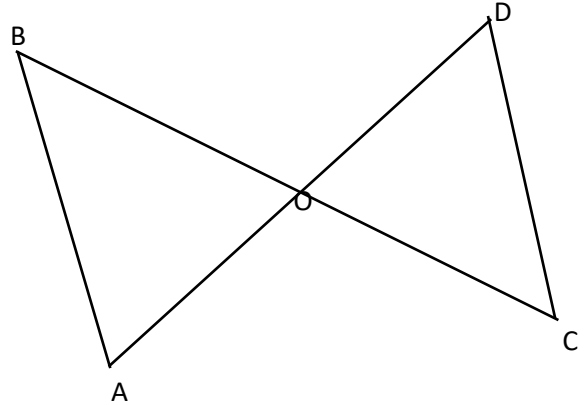
$$\Rightarrow OA = 5.6\text{ cm}$$

$$\text{Taking, } \frac{8}{5} = \frac{6.4\text{ cm}}{OD}$$

$$\Rightarrow 8 \times OD = 5 \times 6.4\text{ cm}$$

$$\Rightarrow OD = \frac{5 \times 6.4^{0.8}\text{ cm}}{8_1}$$

$$\Rightarrow OD = 4\text{ cm}$$



37. In the adjoining figure, $\angle APQ = \angle B$. Prove that $\Delta APQ \sim \Delta ABC$. If $AP = 3.8\text{cm}$, $AQ = 3.6\text{cm}$, $BQ = 2.1\text{cm}$ and $BC = 4.2\text{cm}$ find PQ .

Ans

In ΔABC and ΔAPQ ,

$$\angle APQ = \angle B \quad (\text{Given})$$

$$\angle A = \angle A \quad (\text{Common angle})$$

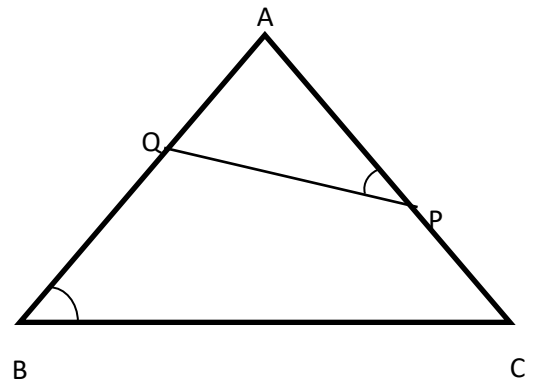
$$\therefore \Delta ABC \sim \Delta APQ \quad (\text{AA Similarity})$$

$$\therefore \frac{AP}{AB} = \frac{PQ}{BC}$$

$$\Rightarrow \frac{3.8}{3.6 + 2.1} = \frac{PQ}{4.2}$$

$$\Rightarrow \frac{3.8}{5.7} = \frac{PQ}{4.2}$$

$$\Rightarrow 5.7 \times PQ = 3.8 \times 4.2\text{ cm}$$

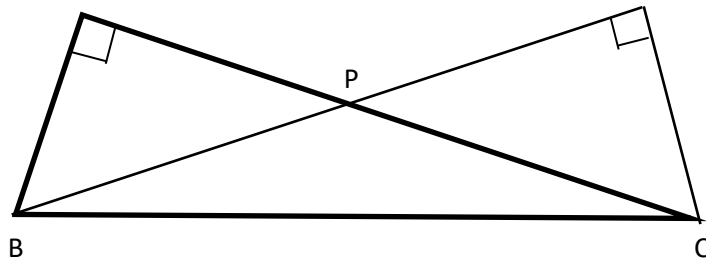


$$\Rightarrow PQ = \frac{3.8^{0.2} \times 4.2^{1.4}}{5.7^{0.3} 0.1} \text{ cm}$$

$$\Rightarrow PQ = 2.8 \text{ cm}$$

38. Two Δs BAC and BDC , right angled at A and D respectively are drawn on the same base BC and on the same side of BC . If AC and DB intersect at P , prove that $AP \times PC = DP \times PB$.

Ans



In Δs PAB and PDC

$$\angle A = \angle D \quad (\text{Each } 90^\circ)$$

$$\angle APB = \angle DPC \quad (\text{Vertically opposite angles})$$

$$\therefore \Delta PAB \sim \Delta PDC \quad (\text{AA Similarity})$$

$$\therefore \frac{AP}{DP} = \frac{PB}{PC}$$

$$\Rightarrow AP \times PC = DP \times PB$$

Hence Proved

39. X and Y are points on side PQ and PR of a ΔPQR . If $PX = 2\text{cm}$, $XQ = 4\text{cm}$ and $XY \parallel QR$, prove that $3XY = QR$.

Ans

In ΔPXY and ΔPQR

$$\angle P = \angle P \quad (\text{Common angle})$$

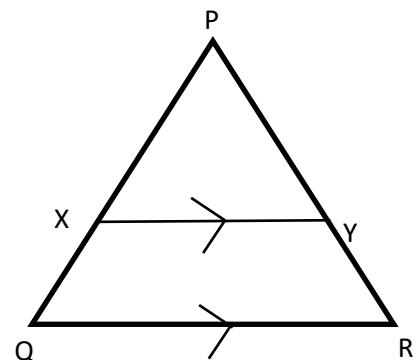
$$\angle Q = \angle PXY \quad (\because XY \parallel QR)$$

$$\therefore \Delta PXY \sim \Delta PQR \quad (\text{AA Similarity})$$

$$\therefore \frac{PX}{PQ} = \frac{XY}{QR}$$

$$\Rightarrow \frac{PX}{PX + XQ} = \frac{XY}{QR}$$

$$\Rightarrow \frac{2}{2+4} = \frac{XY}{QR}$$



$$\Rightarrow \frac{2}{6} = \frac{XY}{QR}$$

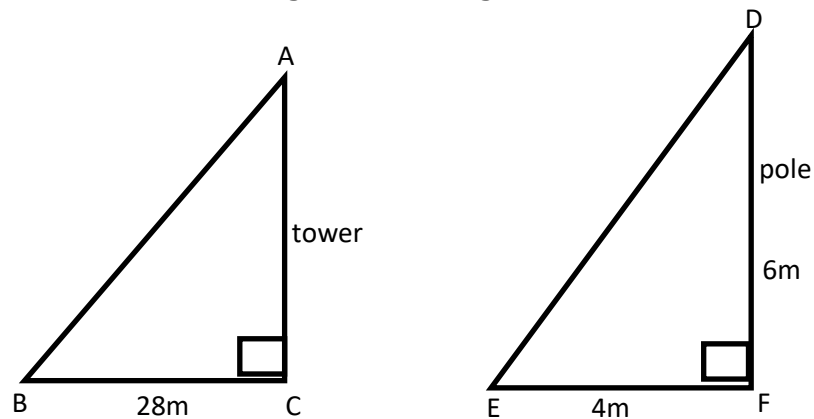
$$\Rightarrow \frac{1}{3} = \frac{XY}{QR}$$

$$\Rightarrow 3XY = QR$$

Hence Proved

40. A vertical tower of length 6m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28m long. Find the height of the tower.

Ans



Let AC be the height of the tower and BC = 28 m be the length of its shadow.

Let DF = 6 m be height of the pole and EF = 4 m be the length of its shadow.

In $\triangle ABC$ and $\triangle DEF$,

$\angle B = \angle E$ (same inclination at the same time)

$\angle C = \angle F$ (each 90°)

$\therefore \triangle ABC \sim \triangle DEF$ (AA Similarity)

$$\therefore \frac{AC}{DF} = \frac{BC}{EF}$$

$$\Rightarrow \frac{AC}{6\text{ m}} = \frac{28}{4}$$

$$\Rightarrow AC = \frac{28 \times 6\text{ m}}{4}$$

$$\Rightarrow AC = 42\text{ m}$$

\therefore The height of the tower is 42 m

41. In $\triangle ABC$, right angled at B, AB = 24cm, BC = 7cm, find Sin A.

Ans

By Pythagoras Theorem in ΔABC , we get

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = (24\text{cm})^2 + (7\text{cm})^2$$

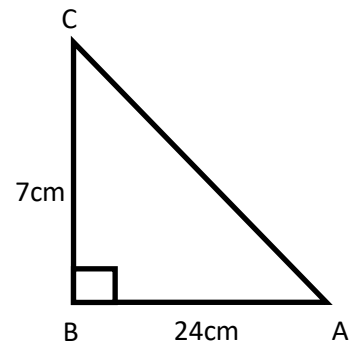
$$\Rightarrow AC^2 = 576\text{cm}^2 + 49\text{cm}^2$$

$$\Rightarrow AC^2 = 625\text{cm}^2$$

$$\Rightarrow AC^2 = \sqrt{625\text{cm}^2}$$

$$\Rightarrow AC^2 = 25\text{cm}$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{7}{25}$$



42. Find the value of $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

Ans

$$\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= \frac{3+1}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

43. Prove that $2\cos^2 30^\circ - 1 = \cos 60^\circ$

Ans

$$\text{LHS} = 2\cos^2 30^\circ - 1$$

$$= 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$$

$$= 2 \times \frac{3}{4} - 1$$

$$= \frac{3}{2} - 1$$

$$= \frac{3-2}{2}$$

$$= \frac{1}{2}$$

$$= \cos 60^\circ$$

$$= \text{RHS}$$

Hence Proved

44. If $A = 60^\circ, B = 30^\circ$, verify that $\cos(A + B) = \cos A \cos B - \sin A \sin B$

Ans L H S = $\cos(A + B)$

$$= \cos(60^\circ + 30^\circ)$$

$$= \cos 90^\circ = 0$$

R H S = $\cos A \cos B - \sin A \sin B$

$$= \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$$

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$$= \frac{\sqrt{3} - \sqrt{3}}{4}$$

$$= \frac{0}{4}$$

$$= 0$$

\therefore L H S = R H S

Hence Verified

45. Prove that $(1 - \sin^2 \theta) \sec^2 \theta = 1$

Ans L H S = $(1 - \sin^2 \theta) \sec^2 \theta$

$$= \cos^2 \theta \times \sec^2 \theta$$

$$= \cancel{\cos^2 \theta} \times \frac{1}{\cancel{\cos^2 \theta}}$$

$$= 1$$

$$= \text{R H S}$$

Hence Proved

46. Prove that $(\sec^2 A - 1)(\operatorname{cosec}^2 A - 1) = 1$

Ans L H S = $(\sec^2 A - 1)(\operatorname{cosec}^2 A - 1)$

$$= \tan^2 A \times \cot^2 A$$

$$= \cancel{\tan^2 A} \times \frac{1}{\cancel{\tan^2 A}}$$

$$= 1$$

= R H S

Hence Proved

47. If $\sin A = \frac{3}{4}$, calculate $\tan A$

Ans Let ABC be a right Δ , right angled at C

$$\sin A = \frac{BC}{BA} = \frac{3}{4}$$

$\therefore BC = 3k$ and $BA = 4k$, where k is a positive number

\therefore By using Pythagoras Theorem, we get

$$AB^2 = BC^2 + AC^2$$

$$\Rightarrow (4k)^2 = (3k)^2 + AC^2$$

$$\Rightarrow 16k^2 = 9k^2 + AC^2$$

$$\Rightarrow 16k^2 - 9k^2 = AC^2$$

$$\Rightarrow 7k^2 = AC^2$$

$$\Rightarrow \sqrt{7k^2} = AC$$

$$\Rightarrow \sqrt{7} k = AC$$

$$\therefore \tan A = \frac{BC}{AC}$$

$$= \frac{3k}{\sqrt{7} k}$$

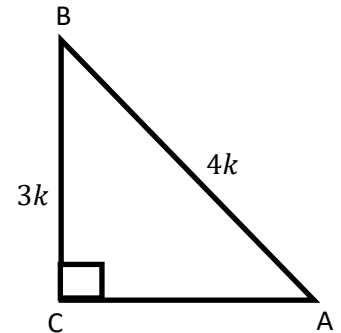
$$= \frac{3k}{\sqrt{7}}$$

48. Find the value of $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$

Ans $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$

$$= \frac{1 - 1^2}{1 + 1^2}$$

$$= \frac{1 - 1}{1 + 1}$$



$$= \frac{0}{2}$$

$$= 0$$

49. Prove that $\frac{1}{1+\tan^2\theta} + \frac{1}{1+\cot^2\theta} = 1$

Ans

$$\begin{aligned} \text{L H S} &= \frac{1}{1+\tan^2\theta} + \frac{1}{1+\cot^2\theta} \\ &= \frac{1}{\sec^2\theta} + \frac{1}{\operatorname{cosec}^2\theta} \quad [\because 1 + \tan^2\theta = \sec^2\theta \text{ and } 1 + \cot^2\theta = \operatorname{cosec}^2\theta] \\ &= \cos^2\theta + \sin^2\theta \\ &= 1 \\ &= \text{R H S} \end{aligned}$$

Hence Proved

50. Express $\frac{1+\tan^2A}{1+\cot^2A}$ in terms of $\tan A$

Ans

$$\begin{aligned} &\frac{1+\tan^2A}{1+\cot^2A} \\ &= \frac{\sec^2A}{\operatorname{cosec}^2A} \\ &= \frac{\frac{1}{\cos^2A}}{\frac{1}{\sin^2A}} \\ &= \frac{1}{\cos^2A} \times \frac{\sin^2A}{1} \\ &= \frac{\sin^2A}{\cos^2A} \\ &= \tan^2A \end{aligned}$$

51. A die is thrown once. What is the probability of getting a number less than 3?

Ans Total number of possible outcomes $\{ie 1, 2, 3, 4, 5, 6\} = 6$

Number of favourable outcomes less than 3 $\{ie 1, 2\} = 2$

$$\begin{aligned} \therefore P(\text{less than } 3) &= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} \\ &= \frac{2}{6} \end{aligned}$$

$$= \frac{1}{3}$$

52. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the possible that the drawn ball is not red.

Ans Total number of possible outcomes = 3 + 5 = 8

Number of favourable outcomes of not red = 5

$$\begin{aligned}\therefore P(\text{not red}) &= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} \\ &= \frac{5}{8}\end{aligned}$$

53. A box contains 20 cards number 1 to 20. A card is drawn at random from the box. Find the probability that the number on the drawn card is a prime number.

Ans Total number of possible outcomes = 20

Prime numbers are 2, 3, 5, 7, 11, 13, 17, 19

\therefore Number of favourable outcomes of a prime number = 8

$$\begin{aligned}\therefore P(\text{prime number}) &= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} \\ &= \frac{8}{20} \\ &= \frac{2}{5}\end{aligned}$$

54. A bag contains lemon flavoured candies only. Maline takes out one candy without looking into the bag. What is the probability that she takes out an orange flavoured candy.

Ans Let the number of lemon flavoured candies be x

\therefore Total number of favourable outcomes = x

Number of orange flavoured candies = 0

$$\begin{aligned}\therefore P(\text{orange flavoured candies}) &= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} \\ &= \frac{0}{x} \\ &= 0\end{aligned}$$

55. A box contains balls numbered 3 to 50. A ball is drawn at random from the box. Find the probability that the drawn ball has a number which is a perfect square.

Ans Total number of possible outcomes = $50 - 3 + 1 = 48$

The perfect squares are 4, 9, 16, 25, 36, 49

\therefore Number of favourable = 6

$$\begin{aligned}\therefore P(\text{perfect square}) &= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} \\ &= \frac{\cancel{6}^1}{\cancel{48}_8} \\ &= \frac{1}{8}\end{aligned}$$

56. One card is drawn at random from a well shuffled deck of 52 cards. Find the probability of getting a king of red colour.

Ans Total number of possible outcomes = 52

Number of favourable outcomes of a king of red colour = 2

$$\begin{aligned}\therefore P(\text{king of red colour}) &= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} \\ &= \frac{\cancel{2}^1}{\cancel{52}_{26}} \\ &= \frac{1}{26}\end{aligned}$$

57. A game of chance consists of spinning an arrow which comes to rest pointing to one of the number 1, 2, 3, 4, 5, 6, 7, 8 and these are equally likely outcomes. What is the probability that it will point greater than 2.

Ans Total number of possible outcomes $\{ie\ 1, 2, 3, 4, 5, 6, 7, 8\} = 8$

Number of favourable outcomes $\{ie\ 3, 4, 5, 6, 7, 8\} = 6$

$$\begin{aligned}\therefore P(\text{number greater than 2}) &= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} \\ &= \frac{\cancel{6}^3}{\cancel{8}_4} \\ &= \frac{3}{4}\end{aligned}$$

58. Five cards-the ten, jack, queen, king and ace of diamonds are well-shuffled with then face downwards. One card is drawn at random. What is the probability that the card drawn is a queen.

Ans Total number of possible outcomes = 5

Number of favourable outcomes = 1

$$\begin{aligned}\therefore P(\text{a queen}) &= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} \\ &= \frac{1}{5}\end{aligned}$$

59. A lot of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that she will buy it.

Ans Total number of ball pens = 144

\therefore Total number of possible outcomes = 144

Number of defective ball pens = 20

\therefore Number of good ball pens = $144 - 20 = 124$

\therefore Number of favourable outcome of good ball pens = 124

$$\begin{aligned}\therefore P(\text{buy it}) &= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} \\ &= \frac{\cancel{124}^{\cancel{62}31}}{\cancel{144}^{\cancel{72}36}} \\ &= \frac{31}{36}\end{aligned}$$

60. A die is thrown twice. What is the probability that 5 will not come up either time.

Ans Total number of possible outcomes = $6 \times 6 = 36$

Number of possible outcomes when 5 will come up either time

{ie (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (1,5), (2,5), (3,5), (4,5), (6,5)} = 11

$$\begin{aligned}\therefore P(\text{5 will come up either time}) &= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} \\ &= \frac{11}{36}\end{aligned}$$

$$\therefore P(\text{5 will not come up either time}) = 1 - \frac{11}{36}$$

$$= \frac{36-11}{36}$$

$$= \frac{25}{36}$$

Section - C
Short Answer Question (3 Marks)

1. Find a quadratic polynomial whose sum and product of zeroes are $\frac{1}{4}$ and -1 respectively.

Soln.: Given: Sum of the zeroes = $\frac{1}{4}$

$$\text{Product of the zeroes} = -1$$

∴ A quadratic polynomial is -

$$x^2 - (\text{sum of the zeroes})x + \text{Product of the zeroes}$$

$$= x^2 - \frac{1}{4}x + (-1)$$

$$= \frac{4x^2 - x - 4}{4}$$

$$= \frac{1}{4}(4x^2 - x - 4)$$

$$\Rightarrow 4x^2 - x - 4, \text{ where } \frac{1}{4} \text{ is a constant.}$$

Hence, the required quadratic polynomial is $4x^2 - x - 4$.

2. Find the zeroes of the quadratic polynomial $10x^2 + 3x - 1$ and verify the relationship between the zeroes and the coefficients.

Soln.: Let $p(x) = 10x^2 + 3x - 1$

$$\begin{aligned} &= 10x^2 + (5 - 2)x - 1 \\ &= 10x^2 + 5x - 2x - 1 \\ &= 5x(2x + 1) - 1(2x + 1) \\ &= (2x + 1)(5x - 1) \end{aligned}$$

$$\therefore p(x) = 0$$

$$\Rightarrow (2x + 1)(5x - 1) = 0$$

$$\text{Either, } 2x + 1 = 0 \text{ or, } 5x - 1 = 0$$

$$\Rightarrow x = \frac{-1}{2} \text{ or, } x = \frac{1}{5}$$

∴ The zeroes of $p(x)$ are $\frac{-1}{2}$ and $\frac{1}{5}$

Verification:

$$\begin{aligned} \text{Sum of the zeroes} &= \frac{-1}{2} + \frac{1}{5} \\ &= \frac{-5+2}{10} \\ &= \frac{-3}{10} \\ &= \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2} \end{aligned}$$

$$\begin{aligned} \text{Product of the zeroes} &= \frac{-1}{2} \times \frac{1}{5} \\ &= \frac{-1}{10} \end{aligned}$$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, verified.

3. Find a quadratic polynomial whose zeroes are $\frac{2}{\sqrt{3}}$ and $\frac{-4}{\sqrt{3}}$.

Soln.: Sum of the zeroes $= \frac{2}{\sqrt{3}} + \left(\frac{-4}{\sqrt{3}}\right) = \frac{2-4}{\sqrt{3}} = \frac{-2}{\sqrt{3}}$

Product of the zeroes $= \frac{2}{\sqrt{3}} \times \left(\frac{-4}{\sqrt{3}}\right) = \frac{-8}{3}$

∴ A quadratic polynomial is -

$$x^2 - (\text{sum of the zeroes})x + \text{Product of the zeroes}$$

$$= x^2 - \left(\frac{-2}{\sqrt{3}}\right)x + \left(\frac{-8}{3}\right)$$

$$= x^2 + \frac{2}{\sqrt{3}}x - \frac{8}{3}$$

$$= \frac{3\sqrt{3}x^2 + 6x - 8\sqrt{3}}{3\sqrt{3}}$$

$$= \frac{1}{3\sqrt{3}}(3\sqrt{3}x^2 + 6x - 8\sqrt{3})$$

$$\Rightarrow 3\sqrt{3}x^2 + 6x - 8\sqrt{3}, \text{ where } \frac{1}{3\sqrt{3}} \text{ is a constant.}$$

Hence, the required quadratic polynomial is $3\sqrt{3}x^2 + 6x - 8\sqrt{3}$.

4. One zero of the quadratic polynomial $2x^2 - 8x - m$ is $\frac{5}{2}$. Find the other zero and the value of m .

Soln.: Given, one of the zeroes is $\frac{5}{2}$.

Let the other zero be y .

Now,

$$\text{Sum of the zeroes} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\Leftrightarrow \frac{5}{2} + y = \frac{-(-8)}{2}$$

$$\Leftrightarrow y = \frac{8}{2} - \frac{5}{2}$$

$$\Leftrightarrow y = \frac{8-5}{2}$$

$$\Leftrightarrow y = \frac{3}{2}$$

Also,

$$\text{Product of the zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\Leftrightarrow \frac{5}{2} \times y = \frac{-m}{2}$$

$$\Leftrightarrow \frac{5}{2} \times \frac{3}{2} = \frac{-m}{2}$$

$$\Leftrightarrow \frac{15}{4} = \frac{-m}{2}$$

$$\Leftrightarrow m = \frac{-15 \times 2}{4}$$

$$\Rightarrow m = \frac{-15}{2}$$

Hence, the other zero is $\frac{3}{2}$ and $m = \frac{-15}{2}$.

5. If α and β are zeroes of the polynomial $p(x) = ax^2 + bx + c$, then evaluate $(\alpha - \beta)^2$.

Soln.: Since, α and β are zeroes of the polynomial,

$$p(x) = ax^2 + bx + c$$

$$\therefore \text{Sum of the zeroes} = \frac{-b}{a}$$

$$\Rightarrow \alpha + \beta = \frac{-b}{a}$$

$$\text{And, Product of the zeroes} = \frac{c}{a}$$

$$\Rightarrow \alpha\beta = \frac{c}{a}$$

Now,

$$\begin{aligned} (\alpha - \beta)^2 &= (\alpha + \beta)^2 - 4\alpha\beta \\ &= \left(\frac{-b}{a}\right)^2 - 4 \cdot \frac{c}{a} \\ &= \frac{b^2}{a^2} - \frac{4c}{a} \\ &= \frac{b^2 - 4ac}{a^2} \quad (\text{Ans}) \end{aligned}$$

6. Find the value of 'p' if the quadratic polynomial $p(x) = x^2 + 8x + p$ has zeroes whose difference is 4.

Soln.: Let α and β be the zeroes of $p(x) = x^2 + 8x + p$.

$$\therefore \alpha + \beta = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\Rightarrow \alpha + \beta = \frac{-8}{1}$$

$$\Rightarrow \alpha + \beta = -8 \quad \text{-----(i)}$$

And,

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\Rightarrow \alpha\beta = \frac{p}{1}$$

$$\Rightarrow \alpha\beta = p \quad \text{-----(ii)}$$

Also, By the question, we have

$$\alpha - \beta = 4 \quad \text{-----(iii)}$$

Adding equations (i) and (iii), we have

$$\alpha + \beta + \alpha - \beta = -8 + 4$$

$$\Rightarrow 2\alpha = -4$$

$$\Rightarrow \alpha = -2$$

Putting $\alpha = -2$ in eqn. (iii), we have

$$\alpha - \beta = 4$$

$$\Rightarrow -2 - \beta = 4$$

$$\Rightarrow -\beta = 4 + 2$$

$$\Rightarrow -\beta = 6$$

$$\Rightarrow \beta = -6$$

Putting the values of α and β in eqn. (ii), we get

$$\alpha\beta = p$$

$$\Rightarrow (-2)(-6) = p$$

$$\Rightarrow p = 12$$

Hence, $p = 12$. (Ans)

7. If the zeroes of the quadratic polynomial $p(x) = 3x^2 + (2k - 1)x - 5$ are equal in magnitude but opposite in sign then find the value of k .

Soln.: Given polynomial is-

$$p(x) = 3x^2 + (2k - 1)x - 5$$

Let the zeroes be y and $-y$.

$$\therefore \text{Sum of the zeroes} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\Rightarrow y + (-y) = \frac{-(2k-1)}{3}$$

$$\Rightarrow y - y = \frac{-2k+1}{3}$$

$$\Rightarrow 0 = \frac{-2k+1}{3}$$

$$\Rightarrow 0 \times 3 = -2k + 1$$

$$\Rightarrow 0 = -2k + 1$$

$$\Rightarrow 2k = 1$$

$$\Rightarrow k = \frac{1}{2} \text{ (Ans)}$$

8. If one zero of the polynomial $(a^2 + 9)x^2 + 15x + 6a$ is reciprocal of the other, find the value of a .

Soln.: Given polynomial is $(a^2 + 9)x^2 + 15x + 6a$

Let the other zero be y

$$\therefore \text{One zero} = \frac{1}{y}$$

Now,

$$\text{Product of the zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\Rightarrow y \times \frac{1}{y} = \frac{6a}{a^2+9}$$

$$\Rightarrow 1 = \frac{6a}{a^2+9}$$

$$\begin{aligned}
&\Rightarrow a^2 + 9 = 6a \\
&\Rightarrow a^2 - 6a + 9 = 0 \\
&\Rightarrow a^2 - (3 + 3)a + 9 = 0 \\
&\Rightarrow a^2 - 3a - 3a + 9 = 0 \\
&\Rightarrow a(a - 3) - 3(a - 3) = 0 \\
&\Rightarrow (a - 3)(a - 3) = 0 \\
&\text{Either, } a - 3 = 0 \text{ or, } a - 3 = 0 \\
&\Rightarrow a = 3 \text{ or, } a = 3 \\
&\text{Hence, the value of } a \text{ is } 3. \text{ (Ans)}
\end{aligned}$$

9. Find the sum and product of the zeroes of the quadratic polynomials:

(i) $x^2 - 8$ (ii) $x^2 - (c - ab)x - abc$

Soln.:(i) Given polynomial is: $x^2 - 8$

$$\begin{aligned}
\therefore \text{ Sum of the zeroes} &= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} \\
&= \frac{0}{1} \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\text{And, Product of the zeroes} &= \frac{\text{Constant term}}{\text{Coefficient of } x^2} \\
&= \frac{-8}{1} \\
&= -8
\end{aligned}$$

(ii) Given polynomial is: $x^2 - (c - ab)x - abc$

$$\begin{aligned}
\therefore \text{ Sum of the zeroes} &= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} \\
&= \frac{-\{-(c-ab)\}}{1} \\
&= c - ab
\end{aligned}$$

$$\begin{aligned}
\text{And, Product of the zeroes} &= \frac{\text{Constant term}}{\text{Coefficient of } x^2} \\
&= \frac{-abc}{1} \\
&= -abc \text{ (Ans)}
\end{aligned}$$

10. Find the value of k such that the quadratic polynomial $x^2 - (k + 6)x + 2(2k + 1)$ has sum of the zeroes as half of their product.

Soln.: We have $p(x) = x^2 - (k + 6)x + 2(2k + 1)$

Let α and β be the zeroes of $p(x)$

Then,

$$\alpha + \beta = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$= \frac{-\{-(k+6)\}}{1}$$

$$= k + 6$$

And,

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= \frac{2(2k+1)}{1}$$

$$= 4k + 2$$

But, According to the question, we have

$$\alpha + \beta = \frac{1}{2}(\alpha\beta)$$

$$\Rightarrow k + 6 = \frac{1}{2}(4k + 2)$$

$$\Rightarrow 2k + 12 = 4k + 2$$

$$\Rightarrow 4k - 2k = 12 - 2$$

$$\Rightarrow 2k = 10$$

$$\Rightarrow k = 5 \text{ (Ans).}$$

11. The age of a man is twice the square of the age of his son. Eight years hence, the age of the man will be 4 years more than three times the age of his son. Find their present ages.

Soln.: Let the present age of the son be x years

Then, the age of the man = $2x^2$ years

\therefore 8 years hence, the age of the son = $(x + 8)$ years

& 8 years hence, the age of the man = $(2x^2 + 8)$ years

Now, According to the question, we have

$$\Rightarrow (2x^2 + 8) = 3(x + 8) + 4$$

$$\Rightarrow 2x^2 + 8 = 3x + 24 + 4$$

$$\Rightarrow 2x^2 + 8 = 3x + 28$$

$$\Rightarrow 2x^2 - 3x + 8 - 28 = 0$$

$$\Rightarrow 2x^2 - 3x - 20 = 0$$

$$\Rightarrow 2x^2 - (8 - 5)x - 20 = 0$$

$$\Rightarrow 2x^2 - 8x + 5x - 20 = 0$$

$$\Rightarrow 2x(x - 4) + 5(x - 4) = 0$$

$$\Rightarrow (x - 4)(2x + 5) = 0$$

Either, $x - 4 = 0$ or, $2x + 5 = 0$

$\Rightarrow x = -4$ or, $x = \frac{-5}{2}$ (Rejected as age cannot be negative)

Hence, son's present age = 4 years.

& man's present age = $2 \times 4^2 = 2 \times 16 = 32$ years. (Ans)

12. Find the value of 'k' for which $2x^2 + kx + 3 = 0$ have equal roots.

Soln.: Comparing $2x^2 + kx + 3 = 0$ with $ax^2 + bx + c = 0$, we have

$$a = 2, b = k, \text{ and } c = 3$$

We know, for equal roots,

$$\text{Discriminant, } D = 0$$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow k^2 - 4 \times 2 \times 3 = 0$$

$$\Rightarrow k^2 - 24 = 0$$

$$\Rightarrow k^2 = 24$$

$$\Rightarrow k = \pm\sqrt{24}$$

$$\Rightarrow k = \pm\sqrt{2 \times 2 \times 6}$$

$$\Rightarrow k = \pm\sqrt{2^2 \times 6}$$

$$\Rightarrow k = \pm 2\sqrt{6}$$

$$\therefore k = 2\sqrt{6}, -2\sqrt{6}. \text{ (Ans)}$$

13. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Soln.: Let the number of pottery articles produced on a particular day be x .

$$\therefore \text{Cost of production of each article} = 2x + 3$$

So, the total cost of production

$$= \text{No. of pottery articles} \times \text{cost of production of each article}$$

$$= x(2x + 3)$$

Now, According to the question, we have

$$x(2x + 3) = 90$$

$$\Rightarrow 2x^2 + 3x = 90$$

$$\Rightarrow 2x^2 + 3x - 90 = 0$$

$$\Rightarrow 2x^2 + (15 - 12)x - 90 = 0$$

$$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$$

$$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$$

$$\Rightarrow (2x + 15)(x - 6) = 0$$

$$\text{Either, } 2x + 15 = 0 \quad \text{or, } x - 6 = 0$$

$$\Rightarrow x = \frac{-15}{2} \quad \text{or, } x = 6$$

But, $x = \frac{-15}{2}$ is rejected as number of articles produced cannot be negative.

Hence, the number of articles produced is 6.

& The cost of each article is $= 2 \times 6 + 3 = 12 + 3 = \text{Rs } 15$. (Ans)

14. The product of two consecutive positive integers is 240. Find the integers.

Soln.: Let the two consecutive positive integers be x and $(x + 1)$.

According to the question, we have

Product of two consecutive positive integers $= 240$

$$\Rightarrow x(x + 1) = 240$$

$$\Rightarrow x^2 + x = 240$$

$$\Rightarrow x^2 + x - 240 = 0$$

$$\Rightarrow x^2 + (16 - 15)x - 240 = 0$$

$$\Rightarrow x^2 + 16x - 15x - 240 = 0$$

$$\Rightarrow x(x + 16) - 15(x + 16) = 0$$

$$\Rightarrow (x + 16)(x - 15) = 0$$

$$\text{Either, } x + 16 = 0 \quad \text{or, } x - 15 = 0$$

$$\Rightarrow x = -16 \quad \text{or, } x = 15$$

Since x is a natural number,

$\therefore x = -16$ is rejected

Hence, the two consecutive positive integers are $x = 15$ & $x + 1 = 15 + 1 = 16$.

15. Find the positive value of k for which $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have real roots.

Soln.: Since, the equation $x^2 + kx + 64 = 0$ has real roots.

\therefore Discriminant ≥ 0

$$\Rightarrow b^2 - 4ac \geq 0$$

$$\Rightarrow k^2 - 4 \times 1 \times 64 \geq 0$$

$$\Rightarrow k^2 - 256 \geq 0$$

$$\Rightarrow k^2 \geq 256$$

$$\Rightarrow k \geq \pm\sqrt{256}$$

$$\Rightarrow k \geq \pm 16$$

$$\Rightarrow k \geq 16 \quad \text{or, } k \leq -16 \quad \text{-----(i)}$$

Also,

Since, the equation $x^2 - 8x + k = 0$ has real roots.

\therefore Discriminant ≥ 0

$$\begin{aligned}
\Rightarrow b^2 - 4ac &\geq 0 \\
\Rightarrow (-8)^2 - 4 \times 1 \times k &\geq 0 \\
\Rightarrow 64 - 4k &\geq 0 \\
\Rightarrow 64 &\geq 4k \\
\Rightarrow 4k &\leq 64 \\
\Rightarrow k &\leq \frac{64}{4} \\
\Rightarrow k &\leq 16 \text{ -----(ii)}
\end{aligned}$$

From equations (i) and(ii) , we have

$$k = 16. \text{ (Ans)}$$

16. Find two numbers whose sum is 27 and the product is 182.

Soln.: Let the first number be x and the second number be $(27 - x)$.

According to the question, we have

Product of two numbers = 182

$$\begin{aligned}
\Rightarrow x(27 - x) &= 182 \\
\Rightarrow 27x - x^2 &= 182 \\
\Rightarrow x^2 - 27x + 182 &= 0 \\
\Rightarrow x^2 - (14 + 13)x + 182 &= 0 \\
\Rightarrow x^2 - 14x - 13x + 182 &= 0 \\
\Rightarrow x(x - 14) - 13(x - 14) &= 0 \\
\Rightarrow (x - 14)(x - 13) &= 0
\end{aligned}$$

Either, $x - 14 = 0$ or, $x - 13 = 0$

$\Rightarrow x = 14$ or, $x = 13$

\therefore First number is equal to 13 or 14.

And, Second number = $27 - x = 27 - 13 = 14$.

Or, Second number = $27 - x = 27 - 14 = 13$.

Hence, the two numbers are 13 and 14.

17. The sum of two numbers is 16. The sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

Soln.: Let one number be x

And the other number = $16 - x$

According to the question, we have

$$\begin{aligned}
\frac{1}{x} + \frac{1}{16-x} &= \frac{1}{3} \\
\Rightarrow \frac{16-x+x}{x(16-x)} &= \frac{1}{3} \\
\Rightarrow \frac{16}{16x-x^2} &= \frac{1}{3}
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow 16x - x^2 = 48 \\
&\Rightarrow x^2 - 16x + 48 = 0 \\
&\Rightarrow x^2 - (12 + 4)x + 48 = 0 \\
&\Rightarrow x^2 - 12x - 4x + 48 = 0 \\
&\Rightarrow x(x - 12) - 4(x - 12) = 0 \\
&\Rightarrow (x - 12)(x - 4) = 0 \\
&\text{Either, } x - 12 = 0 \quad \text{or, } x - 4 = 0 \\
&\Rightarrow x = 12 \quad \text{or, } x = 4 \\
&\therefore \text{The numbers are 4 and 12.}
\end{aligned}$$

18. The sum of a number and its reciprocal is $\frac{17}{4}$. Find the number.

Soln.: Let the number be x .

According to the question, we have

$$\begin{aligned}
x + \frac{1}{x} &= \frac{17}{4} \\
\Rightarrow \frac{x^2 + 1}{x} &= \frac{17}{4} \\
\Rightarrow 4x^2 + 4 &= 17x \\
\Rightarrow 4x^2 - 17x + 4 &= 0 \\
\Rightarrow 4x^2 - (16 + 1)x + 4 &= 0 \\
\Rightarrow 4x^2 - 16x - x + 4 &= 0 \\
\Rightarrow 4x(x - 4) - 1(x - 4) &= 0 \\
\Rightarrow (x - 4)(4x - 1) &= 0 \\
\text{Either, } x - 4 = 0 \text{ or, } 4x - 1 &= 0 \\
\Rightarrow x = 4 \quad \text{or, } x = \frac{1}{4} & \text{ (Rejected as it is a fraction)} \\
\therefore \text{The required number is 4. (Ans)}
\end{aligned}$$

19. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Soln.: Let the uniform speed of the train be x km/hr

Distance travelled by the train = 360 km

$$\therefore \text{Time taken to cover 360 km} = \frac{\text{Distance}}{\text{Speed}} = \frac{360}{x} \text{ hours}$$

If the speed had been 5 km/hr more i.e., $(x + 5)$ km/hr, then

$$\therefore \text{New Time taken to cover 360 km} = \frac{\text{Distance}}{\text{New Speed}} = \frac{360}{x+5} \text{ hours}$$

Now, According to the question, we have

$$\begin{aligned}
\frac{360}{x} - \frac{360}{x+5} &= 1 \\
\Rightarrow \frac{360(x+5) - 360x}{x(x+5)} &= 1
\end{aligned}$$

$$\Rightarrow \frac{360x+1800-360x}{x^2+5x} = 1$$

$$\Rightarrow 1800 = x^2 + 5x$$

$$\Rightarrow x^2 + 5x - 1800 = 0$$

$$\Rightarrow x^2 + (45 - 40)x - 1800 = 0$$

$$\Rightarrow x^2 + 45x - 40x - 1800 = 0$$

$$\Rightarrow x(x + 45) - 40(x + 45) = 0$$

$$\Rightarrow (x + 45)(x - 40) = 0$$

$$\text{Either, } x + 45 = 0 \quad \text{or, } x - 40 = 0$$

$$\Rightarrow x = -45 \quad \text{or, } x = 40$$

($x = -45$ is rejected because speed of train cannot be negative)

$$\therefore x = 40$$

Hence, the required speed of the train is 40 km/hr . (Ans)

20. Find the roots of the quadratic equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ by factorisation.

Soln.: $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$$\Rightarrow \sqrt{2}x^2 + (5 + 2)x + 5\sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$$

$$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$$

$$\Rightarrow (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$$

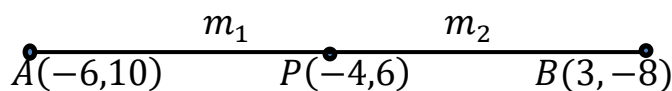
$$\text{Either, } \sqrt{2}x + 5 = 0 \quad \text{or, } x + \sqrt{2} = 0$$

$$\Rightarrow x = \frac{-5}{\sqrt{2}} \quad \text{or, } x = -\sqrt{2}$$

Hence, the roots are $\frac{-5}{\sqrt{2}}$ and $-\sqrt{2}$ (Ans)

21. In which ratio does the point $(-4,6)$ divide the line segment joining the points $A(-6,10)$ and $B(3,-8)$.

Soln.:



Let $P(-4,6)$ divide AB internally in the ratio $m_1 : m_2$

Using the Section Formula, we get

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right)$$

$$\Rightarrow (-4, 6) = \left(\frac{m_1 \times 3 + m_2(-6)}{m_1 + m_2}, \frac{m_1(-8) + m_2 \times 10}{m_1 + m_2} \right)$$

$$\Rightarrow (-4, 6) = \left(\frac{3m_1 - 6m_2}{m_1 + m_2}, \frac{-8m_1 + 10m_2}{m_1 + m_2} \right)$$

$$\text{So, } -4 = \frac{3m_1 - 6m_2}{m_1 + m_2} \quad \text{and} \quad 6 = \frac{-8m_1 + 10m_2}{m_1 + m_2}$$

$$\text{Now, } -4 = \frac{3m_1 - 6m_2}{m_1 + m_2}$$

$$\Rightarrow -4m_1 - 4m_2 = 3m_1 - 6m_2$$

$$\Rightarrow 3m_1 + 4m_1 = 6m_2 - 4m_2$$

$$\Rightarrow 7m_1 = 2m_2$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{2}{7}$$

$$\Rightarrow m_1 : m_2 = 2 : 7$$

Hence, the required ratio is 2 : 7. (Ans)

22. If the points $A(6,1)$, $B(8,2)$, $C(9,4)$ and $D(P,3)$ are the vertices of a parallelogram taken in order, find the value of P .

Soln.: Given, points $A(6,1)$, $B(8,2)$, $C(9,4)$ and $D(P,3)$ are the vertices of a parallelogram $ABCD$.

Since, the diagonals AC and BD of a parallelogram $ABCD$ bisect each other at a point M .

\therefore Mid-point M of AC and mid-point M of BD are same point.

So, Coordinates of mid-point of AC = Coordinates of the mid-point of BD

$$\Rightarrow \left(\frac{6+9}{2}, \frac{1+4}{2} \right) = \left(\frac{8+P}{2}, \frac{2+3}{2} \right)$$

$$\Rightarrow \left(\frac{15}{2}, \frac{5}{2} \right) = \left(\frac{8+P}{2}, \frac{5}{2} \right)$$

$$\text{So, } \frac{15}{2} = \frac{8+P}{2}$$

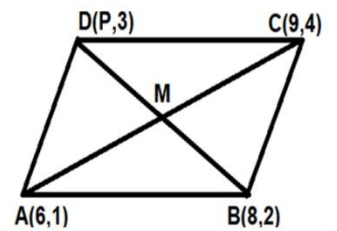
$$\Rightarrow 2(8+P) = 2 \times 15$$

$$\Rightarrow 8+P = \frac{30}{2}$$

$$\Rightarrow 8+P = 15$$

$$\Rightarrow P = 15 - 8$$

$$\Rightarrow P = 7. \text{ (Ans)}$$



23. Find a relation between x and y such that the point (x,y) is equidistant from the points $(7,1)$ and $(3,5)$.

Soln.: Let $P(x,y)$ be equidistant from the points $A(7,1)$ and $B(3,5)$.

$$AP = BP$$

$$\Rightarrow AP^2 = BP^2 \text{ (Squaring both sides)}$$

$$\Rightarrow (x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$$

$$\Rightarrow x^2 - 2 \cdot x \cdot 7 + 7^2 + y^2 - 2 \cdot y \cdot 1 + 1^2 = x^2 - 2 \cdot x \cdot 3 + 3^2 + y^2 - 2 \cdot y \cdot 5 + 5^2$$

$$\Rightarrow x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$$

$$\Rightarrow x^2 - 14x + y^2 - 2y + 50 = x^2 - 6x + y^2 - 10y + 34$$

$$\Rightarrow x^2 - x^2 + y^2 - y^2 - 14x + 6x - 2y + 10y = 34 - 50$$

$$\Rightarrow -8x + 8y = -16$$

$$\Rightarrow -8(x - y) = -16$$

$$\Rightarrow x - y = \frac{-16}{-8}$$

$$\Rightarrow x - y = 2$$

which is the required equation. (Ans)

24. Find a point on the y - axis which is equidistant from the points $(2,3)$ and $(-4,1)$

Soln.: We know that a point on the y - axis is of the form $(0, y)$.

So, let the point $P(0, y)$ be equidistant from $A(2, 3)$ and $B(-4, 1)$.

$$\therefore PA = PB$$

$$\Rightarrow PA^2 = PB^2 \text{ (Squaring both sides)}$$

$$\Rightarrow (2 - 0)^2 + (3 - y)^2 = (-4 - 0)^2 + (1 - y)^2$$

$$\Rightarrow 2^2 + 3^2 - 6y + y^2 = 16 + 1 - 2y + y^2$$

$$\Rightarrow 13 - 6y + y^2 - y^2 = 17 - 2y$$

$$\Rightarrow -6y + 2y = 17 - 13$$

$$\Rightarrow -4y = 4$$

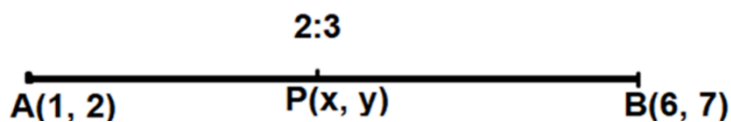
$$\Rightarrow y = \frac{4}{-4}$$

$$\Rightarrow y = -1$$

Hence, the required point on the y - axis is $(0, -1)$. (Ans)

25. If A and B are $(1, 2)$ and $(6, 7)$, respectively, find the coordinates of P such that $AP = \frac{2}{5}AB$ and P lies on the line segment AB .

Soln.:



$$\text{Given, } AP = \frac{2}{5}AB$$

$$\Rightarrow \frac{AB}{AP} = \frac{5}{2}$$

$$\Rightarrow \frac{AP+PB}{AP} = \frac{2+3}{2}$$

$$\Rightarrow 1 + \frac{PB}{AP} = 1 + \frac{3}{2}$$

$$\Rightarrow \frac{PB}{AP} = \frac{3}{2}$$

$$\Rightarrow \frac{AP}{PB} = \frac{2}{3}$$

Let $P(x, y)$ be the point which divides the line segment joining the points $A(1, 2)$ and $B(6, 7)$ in the ratio $2:3$

\therefore BY Section Formula, we get

$$\begin{aligned}
(x, y) &= \left(\frac{2 \times 6 + 3 \times 1}{2 + 3}, \frac{2 \times 7 + 3 \times 2}{2 + 3} \right) \\
&= \left(\frac{12+3}{5}, \frac{14+6}{5} \right) \\
&= \left(\frac{15}{5}, \frac{20}{5} \right) \\
&= (3, 4)
\end{aligned}$$

Hence, the required coordinates of the point P are $(3, 4)$. (Ans)

26. Check whether the points $(3, 0)$, $(6, 4)$ and $(-1, 3)$ are the vertices of an isosceles triangle.

Soln.: Let $A(3, 0)$, $B(6, 4)$ and $C(-1, 3)$ be the given points.

Then, Using Distance Formula, we get

$$AB = \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

$$BC = \sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{(-7)^2 + (-1)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2} \text{ units}$$

$$CA = \sqrt{(-1-3)^2 + (3-0)^2} = \sqrt{(-4)^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5 \text{ units}$$

Since, $AB = CA$

$\therefore \triangle ABC$ is an isosceles triangle. (Ans)

27. Find the values of a for which the distance between the points $P(a, -1)$ and $Q(5, 3)$ is 5 units.

Soln.: According to the question, we have

$$PQ = 5$$

$$\Rightarrow PQ^2 = 5^2 \text{ (Squaring both sides)}$$

$$\Rightarrow (5-a)^2 + (3+1)^2 = 25$$

$$\Rightarrow 5^2 - 2 \cdot 5 \cdot a + a^2 + 4^2 = 25$$

$$\Rightarrow 25 - 10a + a^2 + 16 = 25$$

$$\Rightarrow 25 - 25 - 10a + a^2 + 16 = 0$$

$$\Rightarrow a^2 - 10a + 16 = 0$$

$$\Rightarrow a^2 - (8+2)a + 16 = 0$$

$$\Rightarrow a^2 - 8a - 2a + 16 = 0$$

$$\Rightarrow a(a-8) - 2(a-8) = 0$$

$$\Rightarrow (a-8)(a-2) = 0$$

Either, $a-8=0$ or, $a-2=0$

$$\Rightarrow a=8 \quad \text{or,} \quad a=2$$

$\therefore a=2$ or 8 . (Ans)

28. Find the coordinates of a point A , where AB is the diameter of a circle whose centre is $(2, -1)$ and B is $(5, 7)$.

Soln.: Suppose, AB be the diameter of a circle having its centre at $C(2, -1)$ and

Coordinates of end-point B is $(5, 7)$.

Let the coordinates of A be (x, y) .

Since, AB is the diameter.

$\therefore C$ is the mid-point of AB .

Using Mid-Point Formula,

The coordinates of C are $\left(\frac{x+5}{2}, \frac{y+7}{2}\right)$

But, it is given that the coordinates of C are $(2, -1)$

$$\therefore \frac{x+5}{2} = 2$$

$$\Rightarrow x + 5 = 4$$

$$\Rightarrow x = 4 - 5$$

$$\Rightarrow x = -1$$

And,

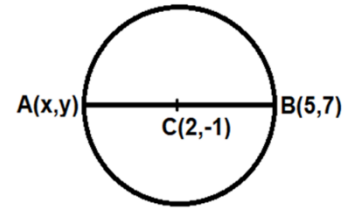
$$\frac{y+7}{2} = -1$$

$$\Rightarrow y + 7 = -2$$

$$\Rightarrow y = -2 - 7$$

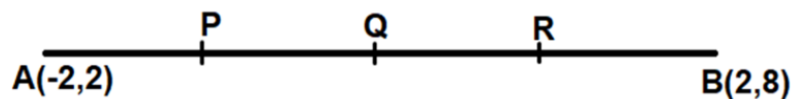
$$\Rightarrow y = -9$$

Hence, the required coordinates of A are $(-1, -9)$. (Ans)



29. Find the coordinates of the points which divide the line segment joining $A(-2, 2)$ and $B(2, 8)$ into four equal parts.

Soln.: Let P, Q and R be the points that divide the line segment joining $A(-2, 2)$ and $B(2, 8)$ into four equal parts.



Since, Q divides the line segment AB into two equal parts i.e., Q is the mid-point of AB .

$$\begin{aligned}\therefore \text{Coordinates of } Q &= \left(\frac{-2+2}{2}, \frac{2+8}{2}\right) \\ &= \left(\frac{0}{2}, \frac{10}{2}\right) \\ &= (0, 5)\end{aligned}$$

Now, P divides the line segment AQ into two equal parts i.e., P is the mid-point of AQ .

$$\therefore \text{Coordinates of } P = \left(\frac{-2+0}{2}, \frac{2+5}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{7}{2} \right)$$

$$= \left(-1, \frac{7}{2} \right)$$

Again, R divides the line segment QB into two equal parts i.e., R is the mid-point of QB .

$$\therefore \text{Coordinates of } R = \left(\frac{0+2}{2}, \frac{5+8}{2} \right)$$

$$= \left(\frac{2}{2}, \frac{13}{2} \right)$$

$$= \left(1, \frac{13}{2} \right)$$

Hence, the required coordinates of the points that divide the line segment AB into four equal parts are $P(0, 5)$, $Q\left(-1, \frac{7}{2}\right)$ and $R\left(1, \frac{13}{2}\right)$. (Ans)

30. Find the coordinates of the point which divides the line segment joining the points $(4, -3)$ and $(8, 5)$ in the ratio 3:1 internally.

Soln.: Let the coordinates of the point be $P(x, y)$.

$$\text{Here, } x_1 = 4, \quad y_1 = -3$$

$$x_2 = 8, \quad y_2 = 5$$

$$m_1 = 3, \quad m_2 = 1$$

\therefore By Section Formula, we have

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{3 \times 8 + 1 \times 4}{3 + 1} = \frac{24 + 4}{4} = \frac{28}{4} = 7$$

And,

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{3 \times 5 + 1 \times (-3)}{3 + 1} = \frac{15 - 3}{4} = \frac{12}{4} = 3$$

Hence, the required coordinates of the point are $P(7, 3)$. (Ans)

31. From a point Q , the length of the tangent to a circle is 24cm and the distance of from the centre is Find the diameter of the circle.

Soln.: Let the length of the tangent be

$$PQ = 24 \text{ cm and } OQ = 25 \text{ cm.}$$

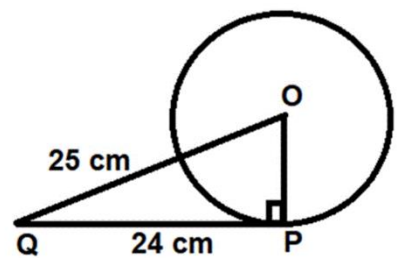
Radius OP is joined.

We know that a tangent at any point of a circle is Perpendicular to the radius through the point of Contact.

$$\therefore OP \perp PQ = 90^\circ$$

Now, in rt. ΔOPQ ,

By Pythagoras theorem, we have



$$OQ^2 = OP^2 + PQ^2$$

$$\Rightarrow (25)^2 = OP^2 + (24)^2$$

$$\Rightarrow 625 = OP^2 + 576$$

$$\Rightarrow OP^2 = 625 - 576$$

$$\Rightarrow OP^2 = 49$$

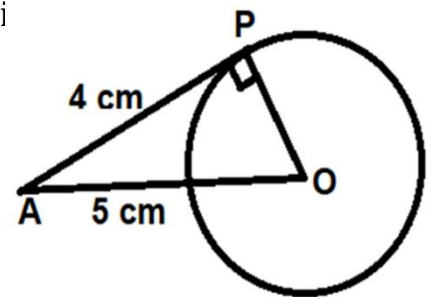
$$\Rightarrow OP = \sqrt{49}$$

$$\Rightarrow OP = 7 \text{ cm}$$

Hence, the required diameter of the circle = $2 \times \text{radius}$
 $= 2 \times 7$
 $= 14 \text{ cm} . \text{ (Ans)}$

32. The length of a tangent from a point A at a distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Soln.: Let O be the centre of the circle and P be the point of contact.
Tangent, $AP = 4 \text{ cm}$ and $OA = 5 \text{ cm}$.
Radius, OP is joined.



Since, the tangent at any point of a circle is Perpendicular to the radius through the point of Contact.

$$\therefore OP \perp AP \Rightarrow \angle OPA = 90^\circ$$

Now, in rt. ΔOPA ,

By Pythagoras theorem, we have

$$OA^2 = OP^2 + AP^2$$

$$\Rightarrow (5)^2 = OP^2 + (4)^2$$

$$\Rightarrow 25 = OP^2 + 16$$

$$\Rightarrow OP^2 = 25 - 16$$

$$\Rightarrow OP^2 = 9$$

$$\Rightarrow OP = \sqrt{9}$$

$$\Rightarrow OP = 3 \text{ cm}$$

Hence, the radius of the circle is 3 cm. (Ans)

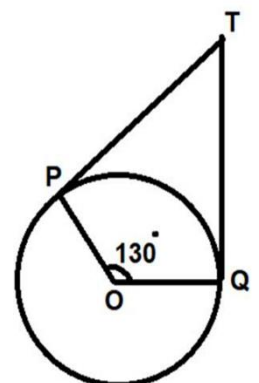
33. In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 130^\circ$, then find $\angle PTQ$.

Soln.: Given: $\angle POQ = 130^\circ$.

We know that a tangent at any point of a circle is Perpendicular to the radius through the point of Contact.

$$\therefore OP \perp PT \text{ and } OQ \perp QT$$

$$\Rightarrow \angle OPT = 90^\circ \text{ and } \angle OQT = 90^\circ \text{ -----(i)}$$



Now, in a quadrilateral $OPTQ$,

Since, the sum of the four angles of a quadrilateral is 360°

$$\therefore \angle POQ + \angle OPT + \angle PTQ + \angle OQT = 360^\circ$$

$$\Rightarrow 130^\circ + 90^\circ + \angle PTQ + 90^\circ = 360^\circ$$

$$\Rightarrow 310^\circ + \angle PTQ = 360^\circ$$

$$\Rightarrow \angle PTQ = 360^\circ - 310^\circ$$

$$\Rightarrow \angle PTQ = 50^\circ \text{ (Ans)}$$

34. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Soln.: Let PQ be the diameter of a given circle with centre O and two tangents AB and CD are drawn to the circle at points P and Q respectively.

Now, since the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\therefore OP \perp AB \text{ and } OQ \perp CD$$

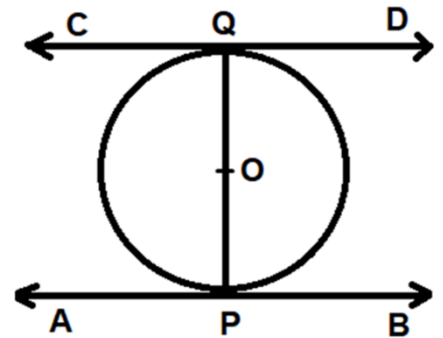
$$\therefore PQ \perp AB \text{ and } PQ \perp CD$$

$$\Rightarrow \angle APQ = 90^\circ \text{ and } \angle PQR = 90^\circ$$

$$\Rightarrow \angle APQ = \angle PQR$$

Since, these are a pair of alternate angles.

$$\therefore AB \parallel CD. \text{ Hence, Proved.}$$



35. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Soln.: Let O be the common centre of two concentric circles and let PQ be a chord of larger circles touching the smaller circles at M .

OM is joined.

$$\text{So, } OM = 3 \text{ cm and } OP = 5 \text{ cm}$$

Since, OM is the radius of the smaller circle

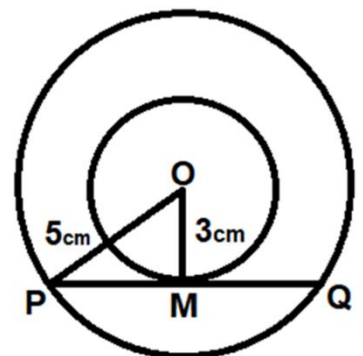
And PQ is the tangent to this circle at M .

$$\therefore OM \perp PQ$$

We know that the perpendicular drawn from the centre of a circle to any chord of the circle, bisects the chord.

i.e., OM bisects PQ

$$\therefore PM = MQ$$



$$\Rightarrow PQ = \frac{1}{2}PQ$$

$$\Rightarrow 2PM = PQ \text{ -----(i)}$$

Now, in rt. ΔPMO ,

By Pythagoras Theorem, we have

$$OP^2 = OM^2 + PM^2$$

$$\Rightarrow (5)^2 = (3)^2 + PM^2$$

$$\Rightarrow 25 = 9 + PM^2$$

$$\Rightarrow PM^2 = 25 - 9$$

$$\Rightarrow PM^2 = 16$$

$$\Rightarrow PM = \sqrt{16}$$

$$\Rightarrow PM = 4 \text{ cm.}$$

\therefore From eqn. (i), we have

$$PQ = 2PM = 2 \times 4 = 8 \text{ cm}$$

Hence, the length of the chord of the larger circle which touches the smaller circle is 8 cm. (Ans)

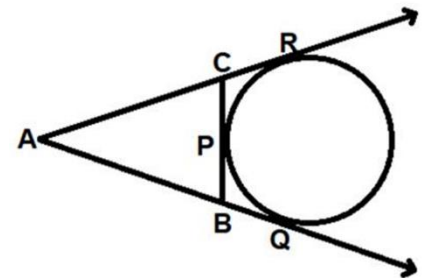
36. A circle touches the side BC of ΔABC , at P and touches AB and AC produced at Q and R respectively. Prove that $AQ = \frac{1}{2}$ (Perimeter of ΔABC).

Soln.: Since the lengths of the tangents drawn from an external point to a circle are equal.

$$\therefore BP = BQ \text{ (Tangents from B) -----(i)}$$

$$CP = CR \text{ (Tangents from C) -----(ii)}$$

$$\& AQ = AR \text{ (Tangents from A) -----(iii)}$$



Now,

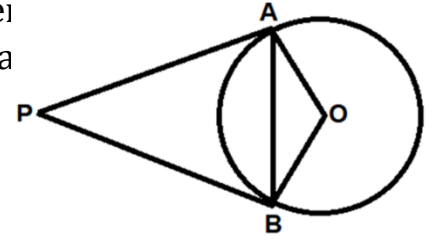
$$\begin{aligned} \text{Perimeter of } \Delta ABC &= AB + BC + AC \\ &= AB + (BP + PC) + AC \\ &= AB + BQ + CR + AC \text{ [from (i)&(ii)]} \\ &= (AB + BQ) + (AC + CR) \\ &= AQ + AR \\ &= AQ + AQ \text{ [from (iii)]} \\ &= 2AQ \end{aligned}$$

$$\therefore AQ = \frac{1}{2}(\text{perimeter of } \Delta ABC)$$

Hence, Proved.

37. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Soln.: Let PA and PB be two tangents drawn from an external point P to a circle with centre O.



To Prove: $\angle AOB + \angle APB = 180^\circ$

Proof: Since, tangent at any point of a circle is Perpendicular to the radius through the point of contact.

$$\therefore OA \perp AP \Rightarrow \angle OAP = 90^\circ \text{ ----- (i)}$$

$$\& OB \perp BP \Rightarrow \angle OBP = 90^\circ \text{ ----- (ii)}$$

Now,

Since, the sum of the four angles of a quadrilateral is 360°

$$\therefore \angle AOB + \angle OAP + \angle APB + \angle OBP = 360^\circ$$

$$\Rightarrow \angle AOB + 90^\circ + \angle APB + 90^\circ = 360^\circ \text{ [from (i) \& (ii)]}$$

$$\Rightarrow \angle AOB + \angle APB + 180^\circ = 360^\circ$$

$$\Rightarrow \angle AOB + \angle APB = 360^\circ - 180^\circ$$

$$\Rightarrow \angle AOB + \angle APB = 180^\circ$$

Hence, Proved.

38. Prove that a parallelogram circumscribing a circle is a rhombus.

Soln.: Let PQRS be a parallelogram such that its sides touch a circle with centre O.

We know, that the lengths of tangents drawn from an external point to a circle are equal.

Therefore, we have

$$PA = PD \text{ (Tangents from P)----- (i)}$$

$$QA = QB \text{ (Tangents from Q)----- (ii)}$$

$$RC = RB \text{ (Tangents from R)----- (iii)}$$

$$\& SC = SD \text{ (Tangents from S)----- (iv)}$$

Now, Adding (i), (ii), (iii) & (iv), we get

$$PA + QA + RC + SC = PD + QD + RB + SD$$

$$\Rightarrow (PA + QA) + (RC + SC) = (PD + SD) + (QB + RB)$$

$$\Rightarrow PQ + RS = PS + QR$$

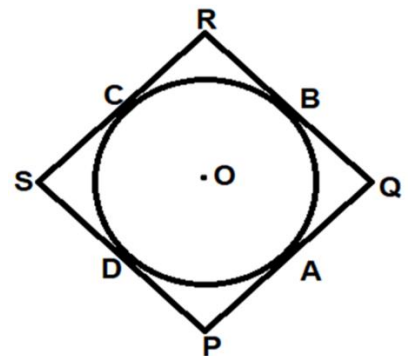
$$\Rightarrow PQ + PQ = QR + QR \text{ [since, PQRS is a parallelogram, } PQ = RS \text{ \& } QR = PS]$$

$$\Rightarrow 2PQ = 2QR$$

$$\Rightarrow PQ = QR$$

$$\therefore PQ = QR = RS = PS$$

Hence, PQRS is a rhombus. (Proved)



39. In the figure, if $PM = 15\text{cm}$, $RL = 12\text{cm}$ and $QN = 7\text{cm}$, then find the perimeter of the triangle PQR .

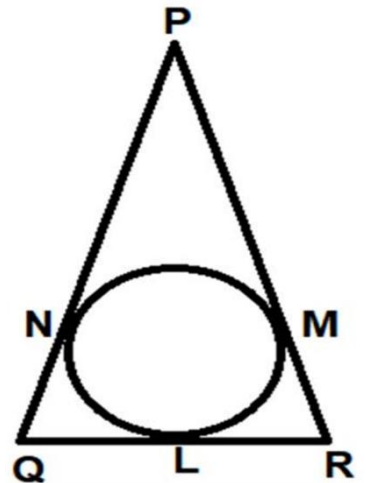
Soln.: Given: $PM = 15\text{cm}$, $RL = 12\text{cm}$
and $QN = 7\text{cm}$

We know, that the lengths of two tangents
Drawn from an external point to a circle
are equal.

$$\begin{aligned} \therefore PM &= PN = 15\text{cm} \\ QN &= QL = 7\text{cm} \\ RL &= RM = 12\text{cm} \end{aligned}$$

Now,

$$\begin{aligned} \text{Perimeter of } \Delta PQR &= PQ + QR + RP \\ &= (PN + QN) + (QL + RL) + (RM + MP) \\ &= (15 + 7) + (7 + 12) + (12 + 15) \\ &= 22 + 19 + 27 \\ &= 68\text{cm (Ans)} \end{aligned}$$



40. In the figure, if $AB = AC$, prove that $BE = EC$.

Soln.: Given: $AB = AC$

To Prove: $BE = EC$

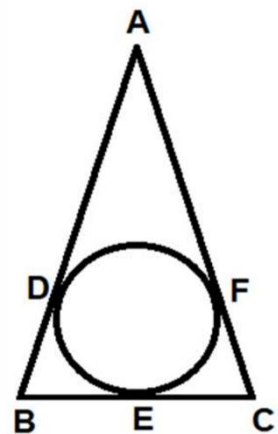
Since, the lengths of two tangents drawn from
an external point to a circle are equal.

$$\begin{aligned} \therefore AD &= AF \text{ (Tangents from A)-----(i)} \\ BD &= BE \text{ (Tangents from B)-----(ii)} \\ CE &= CF \text{ (Tangents from C)-----(iii)} \end{aligned}$$

Now,

$$\begin{aligned} AB &= AC \text{ (given)} \\ \Rightarrow AD + BD &= AF + CF \\ \Rightarrow AF + BD &= AF + CF \text{ [From (i)]} \\ \Rightarrow AF - AF + BD &= CF \\ \Rightarrow BD &= CF \\ \Rightarrow BE &= EC \text{ [From (ii)&(iii)]} \end{aligned}$$

Hence, $BE = EC$. Proved



41. The area of a square is same as the area of a circle. What is the ratio of their perimeters?

Soln.: Let, a be the side of a square
& r be the radius of a circle

Given that, *Area of a square = Area of a circle*

$$\Rightarrow a^2 = \pi r^2$$

$$\Rightarrow a = \sqrt{\pi r^2}$$

$$\Rightarrow a = r\sqrt{\pi}$$

Now,

$$\begin{aligned} \text{Required ratio of their perimeters} &= \frac{\text{Perimeter of a square}}{\text{Perimeter of a circle}} \\ &= \frac{4a}{2\pi r} \\ &= \frac{4(r\sqrt{\pi})}{2\pi r} \\ &= \frac{2\sqrt{\pi}}{\pi} \\ &= \frac{2}{\sqrt{\pi}} \\ &= 2:\sqrt{\pi} \text{ (Ans)} \end{aligned}$$

42. Find the area of the sector of a circle of radius 8 cm and the angle at the centre 30°

Soln.: Given: Radius, $r = 8 \text{ cm}$
& Central angle, $\theta = 30^\circ$

Now,

$$\begin{aligned} \text{Area of sector of a circle} &= \frac{\theta}{360^\circ} (\pi r^2) \\ &= \frac{30^\circ}{360^\circ} \left\{ \frac{22}{7} \times (8)^2 \right\} \\ &= \frac{1}{12} \times \frac{22}{7} \times 8 \times 8 \\ &= \frac{352}{21} \text{ cm}^2 \\ &= 16.76 \text{ cm}^2 \text{ (Ans)} \end{aligned}$$

43. A chord of a circle of radius 14 cm subtends an angle 60° at the centre. Find the area of the major sector.

Soln.: Given: Radius, $r = 14 \text{ cm}$
& Central angle (minor sector) = 60°
 \therefore Central angle for major sector, $\theta = 360^\circ - 60^\circ = 300^\circ$

Now,

$$\begin{aligned} \text{Area of the major sector} &= \frac{\theta}{360^\circ} (\pi r^2) \\ &= \frac{300^\circ}{360^\circ} \left\{ \frac{22}{7} \times (14)^2 \right\} \\ &= \frac{5}{6} \times \frac{22}{7} \times 14 \times 14 \\ &= 154 \text{ cm}^2 \text{ (Ans)} \end{aligned}$$

44. Find the angle subtended at the centre of a circle of radius 5 cm by an arc of length $\frac{5\pi}{3}\text{ cm}$.

Soln.: Given: Radius, $r = 5\text{ cm}$

Let the Central angle be θ .

We Know,

$$\text{Length of an arc} = \frac{\theta}{360^\circ} (2\pi r)$$

$$\Rightarrow \frac{5\pi}{3} = \frac{\theta}{360^\circ} (2 \times \pi \times 5)$$

$$\Rightarrow \frac{5}{3} = \frac{\theta}{360^\circ} \times 10$$

$$\Rightarrow \frac{5}{3} = \frac{\theta}{36}$$

$$\Rightarrow \theta = \frac{5 \times 36}{3}$$

$$\Rightarrow \theta = 60^\circ \text{ (Ans)}$$

45. An arc of length $20\pi\text{ cm}$ subtends an angle 144° at the centre. Find the radius of the circle.

Soln.: Given: Central angle, $\theta = 144^\circ$

& Length of an arc, $l = 20\pi\text{ cm}$

Let, r be the radius of the circle.

Now,

$$\text{Length of an arc} = \frac{\theta}{360^\circ} (2\pi r)$$

$$\Rightarrow 20\pi = \frac{144^\circ}{360^\circ} (2\pi r)$$

$$\Rightarrow 20 = \frac{144}{360} (2r)$$

$$\Rightarrow r = \frac{20 \times 360}{144 \times 2}$$

$$\Rightarrow r = 25\text{ cm} \text{ (Ans)}$$

46. The minute hand of a clock is 7 cm long. Find the area swept by the minute hand between 9 AM to $9:05\text{ AM}$.

Soln.: Given: Radius, $r = 7\text{ cm}$

We know, $60\text{ minutes} = 360^\circ$

$$\Rightarrow 1\text{ minute} = \frac{360^\circ}{60}$$

$$\Rightarrow 5\text{ minutes} = \frac{360}{60} \times 5 = 30^\circ$$

$$\Rightarrow \text{Central angle, } \theta = 30^\circ$$

Now,

$$\begin{aligned} \text{Area swept by the minute hand} &= \text{Area of the minor sector} \\ &= \frac{\theta}{360^\circ} (\pi r^2) \end{aligned}$$

$$\begin{aligned}
&= \frac{30}{360} \left\{ \frac{22}{7} \times (7)^2 \right\} \\
&= \frac{1}{12} \left(\frac{22}{7} \times 7 \times 7 \right) \\
&= \frac{77}{6} \text{ cm}^2 \\
&= 12.83 \text{ cm}^2 \text{ (Ans)}
\end{aligned}$$

47. Find the area of a quadrant of a circle whose circumference is 22 cm.

Soln.: Let, r be the radius of a circle.

Given, Circumference of a circle = 22 cm

$$\Rightarrow 2\pi r = 22$$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 22$$

$$\Rightarrow r = \frac{22 \times 7}{2 \times 22}$$

$$\Rightarrow r = \frac{7}{2} \text{ cm}$$

Now,

$$\begin{aligned}
\text{Area of a quadrant of a circle} &= \frac{1}{4} \pi r^2 \\
&= \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2} \right)^2 \\
&= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \\
&= \frac{77}{8} \text{ cm}^2 \\
&= 9.63 \text{ cm}^2 \text{ (Ans)}
\end{aligned}$$

48. An umbrella has 8 ribs which are equally spaced. Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

Soln.: Given: Radius, $r = 45 \text{ cm}$

Since, there are 8 ribs in the umbrella

$$\therefore \text{Central angle, } \theta = \frac{360}{8} = 45^\circ$$

Now,

Area between two consecutive ribs = Area of the minor sector

$$\begin{aligned}
&= \frac{\theta}{360^\circ} (\pi r^2) \\
&= \frac{45}{360} \left\{ \frac{22}{7} \times (45)^2 \right\} \\
&= \frac{1}{8} \left(\frac{22}{7} \times 45 \times 45 \right) \\
&= \frac{44550}{56} \\
&= 795.5 \text{ cm}^2
\end{aligned}$$

Hence, the area between the two consecutive ribs is 795.5 cm². (Ans)

49. A car has two wipers which do not overlap. Each wiper has a length 25 cm sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades.

Soln.: Given: Radius, $r = 25\text{ cm}$
& Central angle, $\theta = 115^\circ$

Now,

$$\begin{aligned} \text{Area swept by each blade} &= \text{Area of the sector} \\ &= \frac{\theta}{360^\circ} (\pi r^2) \\ &= \frac{115}{360} \left\{ \frac{22}{7} \times (25)^2 \right\} \\ &= \frac{115}{360} \left(\frac{22}{7} \times 25 \times 25 \right) \\ &= \frac{1581250}{2520} \\ &= 627.48\text{ cm}^2 \end{aligned}$$

Hence, total area cleaned at each sweep by 2 blades $= 2 \times 627.48$
 $= 1254.96\text{ cm}^2$ (Ans)

50. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km . Find the area of the sea over which the ships are warned. (use $\pi = 3.14$)

Soln.: Given: Radius, $r = 16.5\text{ km}$
& Central angle, $\theta = 80^\circ$

Now,

$$\begin{aligned} \text{Area of the sector} &= \frac{\theta}{360^\circ} (\pi r^2) \\ &= \frac{80}{360} \{3.14 \times (16.5)^2\} \\ &= \frac{80}{360} (3.14 \times 16.5 \times 16.5) \\ &= \frac{1709.73}{9} \\ &= 189.97\text{ km}^2 \end{aligned}$$

Hence, the area of the sea over which the ships are warned is 189.97 km^2 .
(Ans)

51. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table below. Find the mode of the data:

No. of cars	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	7	14	13	12	20	11	15	8

Soln.: Since the maximum frequency is 20 and the class corresponding to this frequency is 40 – 50.

So, the modal class is 40 – 50.

Therefore, Lower limit (l) of the modal class = 40

Class size (h) = 10

Frequency (f_1) of the modal class = 20

Frequency (f_0) of the class preceding the modal class = 12

Frequency (f_2) of the class succeeding the modal class = 11

Now, Using the formula, we have

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 40 + \left(\frac{20 - 12}{2 \times 20 - 12 - 11} \right) \times 10 \\ &= 40 + \left(\frac{8}{40 - 23} \right) \times 10 \\ &= 40 + \frac{80}{17} \\ &= 40 + 4.7 \\ &= 44.7 \text{ cars (Ans)} \end{aligned}$$

52. The mileage (Km per litre) of 50 cars of the same model was tested by a manufacturer and details are tabulated as given below:

Mileage(Km per litre)	10 – 12	12 – 14	14 – 16	16 – 18
Number of cars	7	12	18	13

Find the mean mileage.

Soln.:

Mileage (Km per litre)	Class Mark (x_i)	Number of cars (f_i)	$f_i x_i$
10 – 12	11	7	77
12 – 14	13	12	156
14 – 16	15	18	270
16 – 18	17	13	221
		$\sum f_i = 50$	$\sum f_i x_i = 724$

Now, Using the formula, we have

$$\begin{aligned} \text{Mean, } \bar{x} &= \frac{\sum f_i x_i}{\sum f_i} \\ &= \frac{724}{50} \\ &= 14.48 \end{aligned}$$

Hence, mean mileage is 14.48 Km per litre. (Ans)

53. The maximum bowling speeds in Km per hour, of 33 players at a cricket coaching centre are given as follows:

Speed (Km per hour)	85 – 100	100 – 115	115 – 130	130 – 145
Number of players	11	9	8	5

Calculate the median bowling speed.

Soln.:

Speed (Km per hour)	Number of players	Cumulative Frequency
85 – 100	11	11
100 – 115	9	11 + 9 = 20
115 – 130	8	20 + 8 = 28
130 – 145	5	28 + 5 = 33

Here, $n = 33$

Now, $\frac{n}{2} = \frac{33}{2} = 16.5$ which lies in the class 100 – 115.

So, the median class is 100 – 115

Here, l = lower limit of the median class = 100

n = number of observation = 33

cf = cumulative frequency of the class preceding the median class =

11

f = frequency of the median class = 9

h = class size = 15

Now, Using the formula, we have

$$\begin{aligned}
 \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\
 &= 100 + \left(\frac{16.5 - 11}{9} \right) \times 15 \\
 &= 100 + \left(\frac{5.5}{9} \right) \times 15 \\
 &= 100 + \frac{82.5}{9} \\
 &= 100 + 9.17 \\
 &= 109.17
 \end{aligned}$$

Hence, median bowling speed is 109.17 Km per hour. (Ans)

54. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality.

Monthly Consumption (in units)	Number of consumers
--------------------------------	---------------------

65 – 85	4
85 – 105	5
105 – 125	13
125 – 145	20
145 – 165	14
165 – 185	8
185 – 205	4

Based on the above information, answer the following questions:

- (i) Find the median class of the data.
- (ii) Find the class mark of the median class.

Soln.:

Monthly Consumption (in units)	Number of consumers	Cumulative Frequency (<i>cf</i>)
65 – 85	4	4
85 – 105	5	4 + 5 = 9
105 – 125	13	9 + 13 = 22
125 – 145	20	22 + 20 = 42
145 – 165	14	42 + 14 = 56
165 – 185	8	56 + 8 = 64
185 – 205	4	64 + 4 = 68
Total	$N = 68$	

Here, $n = 68$

$\Rightarrow \frac{n}{2} = \frac{68}{2} = 34$ which is just greater than the cumulative frequency 22 and lies in the class 125 – 145.

- (i) So, the median class of the given data is 125 – 145.
- (ii) The class mark of the median class 125 – 145 is

$$\begin{aligned}
 &= \frac{\text{lower class limit} + \text{upper class limit}}{2} \\
 &= \frac{125 + 145}{2} \\
 &= \frac{270}{2} \\
 &= 135. \text{ (Ans)}
 \end{aligned}$$

55. For the following distribution:

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	10	15	12	20	9

Find the sum of lower limit of the median class and the lower limit of the modal class.

Soln.:

Class	Frequency	Cumulative Frequency (cf)
0 – 5	10	10
5 – 10	15	10 + 15 = 25
10 – 15	12	25 + 12 = 37
15 – 20	20	37 + 20 = 57
20 – 25	9	57 + 9 = 66

Here, $n = 66$

$\Rightarrow \frac{n}{2} = \frac{66}{2} = 33$, which lies in the class interval 10 – 15.

So, Median class is 10 – 15

Therefore, lower limit of the median class is 10

Also, the highest frequency is 20, which lies in the class interval 15 – 20.

So, Modal Class is 15 – 20

Therefore, lower limit of the modal class is 15

Hence, required sum = 10 + 15 = 25. (Ans)

56. Find the mean of the following data:

x_i	13	15	17	19	21	23
f_i	8	2	3	4	5	6

Soln.:

x_i	f_i	$f_i x_i$
13	8	104
15	2	30
17	3	51
19	4	76
21	5	105
23	6	138
	$\sum f_i = 28$	$\sum f_i x_i = 504$

Now, Using the formula, we have

$$\text{Mean, } \bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{504}{28} = 18. \text{ (Ans)}$$

57. For the following distribution:

Marks	Number of students
<i>Below</i> 10	3
<i>Below</i> 20	12
<i>Below</i> 30	27
<i>Below</i> 40	57
<i>Below</i> 50	75
<i>Below</i> 60	80

Find the modal class.

Soln.:

Marks	Number of students	Cumulative Frequency (<i>cf</i>)
<i>Below</i> 10	3	3
10 – 20	(12 – 3) = 9	12
20 – 30	(27 – 12) = 15	27
30 – 40	(57 – 27) = 30	57
40 – 50	(75 – 57) = 18	75
50 – 60	(80 – 75) = 5	80

So, we see that the highest frequency is 30, which lies in the class interval 30 – 40.

∴ The modal class is 30 – 40. (Ans)

58. What is the difference of median and mean, if the difference of mode and median is 24?

Soln.: Given:

$$\text{Mode} - \text{Median} = 24$$

$$\Rightarrow \text{Mode} = 24 + \text{Median} \text{ -----(i)}$$

Now, Relation among mean, median and mode is –

$$\text{Mode} = 3\text{Median} - 2\text{Mean} \text{ -----(ii)}$$

From equation (i) and (ii), we get

$$3 \text{ Median} - 2 \text{ Mean} = 24 + \text{Median}$$

$$\Rightarrow 3 \text{ Median} - \text{Median} - 2 \text{ Mean} = 24$$

$$\Rightarrow 2 \text{ Median} - 2 \text{ Mean} = 24$$

$$\Rightarrow 2(\text{Median} - \text{Mean}) = 24$$

$$\Rightarrow \text{Median} - \text{Mean} = \frac{24}{2}$$

$$\Rightarrow \text{Median} - \text{Mean} = 12. \text{ (Ans)}$$

59. Find the mode, if

$$l = \text{Lower limit of the modal class} = 1500$$

$$h = \text{Class size} = 500$$

$$f_1 = \text{Frequency of the modal class} = 40$$

f_0 = Frequency of the class preceding the modal class = 24

f_2 = Frequency of the class succeeding the modal class = 33

Soln.: Using the formula, we have

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 1500 + \left(\frac{40 - 24}{2 \times 40 - 24 - 33} \right) \times 500 \\ &= 1500 + \left(\frac{16}{80 - 57} \right) \times 500 \\ &= 1500 + \left(\frac{16}{23} \right) \times 500 \\ &= 1500 + \frac{8000}{23} \\ &= 1500 + 347.83 \\ &= 1847.83 \quad (\text{Ans}) \end{aligned}$$

60. Find the median, if

l = lower limit of the median class = 3000

n = number of observation = 400

cf = cumulative frequency of the class preceding the median class = 130

f = frequency of the median class = 86

h = class size = 500

Soln.: Using the formula, we have

$$\begin{aligned} \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 3000 + \left(\frac{\frac{400}{2} - 130}{86} \right) \times 500 \\ &= 3000 + \left(\frac{200 - 130}{86} \right) \times 500 \\ &= 3000 + \left(\frac{70}{86} \right) \times 500 \\ &= 3000 + \frac{35000}{86} \\ &= 3000 + 406.98 \\ &= 3406.98 \quad (\text{Ans}) \end{aligned}$$

Section- D

Long Answer Questions (5 Marks)

1. The first term of an A.P is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference. Write the first three terms of the AP.

Ans.

Let a_1 be the first term of an AP

Let a_n be the last term of an AP

Let S_n be the sum of the n^{th} term of an AP

Let n be the number of terms of an AP

Let d be the common difference of an AP

Here, $a_1=5$, $a_n=45$, $S_n=400$

Given, $S_n=400$

$$\Rightarrow \frac{n}{2} (a_1 + a_n) = 400$$

$$\Rightarrow \frac{n}{2} (5+45) = 400$$

$$\Rightarrow 50n=400 \times 2$$

$$\Rightarrow n = \frac{400 \times 2}{50}$$

$$n = 16$$

Hence, there are 16 numbers of terms in the given AP.

But, $a_n = 45$

$$\Rightarrow a_1 + (n-1) d = 45$$

$$\Rightarrow 5 + (16-1) d = 45$$

$$\Rightarrow 15d = 45-5$$

$$\Rightarrow 15d = 40$$

$$d = \frac{40}{15} = \frac{8}{3}$$

Hence, the common difference of an AP is $\frac{8}{3}$

The first three terms of the series are: $(a, a + d, a + 2d$ OR, $a_1, a_1 + d, a_2 + d)$

$5, 5 + \frac{8}{3} = 15 + \frac{8}{3} = \frac{23}{3}, 5 + \frac{23}{3} = \frac{38}{3}$ i. e. $5, \frac{23}{3}, \frac{38}{3}$.

2. Find the sum of the first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Solution: Let a_1, a_2, a_3 be the first, second and the third terms of an AP.

Let 'd' be the common difference of an AP.

Here $a_2 = 14$ and $a_3 = 18$, $n = 51$

Now, $a_2 = 14$

$$a_1 + d = 14 \dots\dots\dots (i)$$

and, $a_3 = 18$

$$a_2 + d = 18 \dots\dots\dots (ii)$$

Subtracting equation (i) from equation (ii) we get

$$d = 4$$

Putting $d = 4$ in equation (i) we get

$$a_1 + 4 = 14$$

$$\Rightarrow a_1 = 14 - 4$$

$$\Rightarrow a_1 = 10$$

Using the formula

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_{51} = \frac{n}{2} \{a_1 + a_1 + (n-1) d\} \quad \text{where } a_n = a + (n-1)$$

d

$$= \frac{51}{2} \{2 \times 10 + (51-1) \times 4\}$$

$$= \frac{51}{2} \{20 + 200\}$$

$$= \frac{51}{2} \times 220$$

$$= 5610$$

Hence the sum of the first 51 term of an AP is 5610

3. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution:

Let a_1 be the first term of an AP

Let d be the common difference of an AP

Given, $S_7 = 49$

$$\Rightarrow \frac{7}{2} (a_1 + a_7) = 49 \quad [\because S_n = \frac{n}{2} (a_1 + a_n)]$$

$$\Rightarrow a_1 + a_1 + 6d = \frac{49 \times 2}{7}$$

$$\Rightarrow 2a_1 + 6d = 14$$

$$\Rightarrow a_1 + 3d = 7 \dots\dots\dots (i) \quad (\text{by dividing each term by 2})$$

And, $S_{17} = 289$

$$\frac{17}{2} (a_1 + a_{17}) = 289 [\because S_n = \frac{n}{2} (a_1 + a_n)]$$

$$\Rightarrow a_1 + a_1 + 16d = \frac{289 \times 2}{17}$$

$$\Rightarrow 2a_1 + 16d = 34$$

$$\Rightarrow a_1 + 8d = 17 \dots\dots\dots (ii) \text{ [By dividing each term by 2]}$$

Subtracting equation (i) from equation (ii) we get

$$5d = 10$$

$$\Rightarrow d = \frac{10}{5}$$

$$\Rightarrow d = 2$$

Putting $d = 2$ in equation (i) we get

$$a_1 + 3 \times 2 = 7$$

$$\Rightarrow a_1 = 7 - 6$$

$$\Rightarrow a_1 = 1$$

Using the formula

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$= \frac{n}{2} \{2a_1 + (n-1)d\}$$

$$= \frac{n}{2} \{2 \times 1 + (n-1) \times d\}$$

$$= \frac{n}{2} \{2 + 2(n-1)\}$$

$$= \frac{2n}{2} (1 + n - 1)$$

$$= n \times n$$

$$S_n = n^2$$

Hence, sum of first n terms is n^2

4. How many three- digit numbers are divisible by 7?

Solution: The first three-digit number divisible by 7 is 105

And the last three-digit number divisible by 7 is 994

Therefore, the AP is 105, 112, 994

Here, First term, $a_1 = 105$, second term, $a_2 = 112$

Common difference, $d = 7$

Let n be the number terms of the given AP

Now, $a_n = 994$

$$\begin{aligned} \Rightarrow a_1 + (n-1)d &= 994 \\ \Rightarrow 105 + (n-1) \times 7 &= 994 \\ \Rightarrow (n-1) \times 7 &= 994 - 105 \\ \Rightarrow (n-1) \times 7 &= 889 \\ \Rightarrow n - 1 &= \frac{889}{7} \\ \Rightarrow n - 1 &= 127 \\ \Rightarrow n &= 127 + 1 \\ \Rightarrow n &= 128 \end{aligned}$$

Hence there are 128 three-digit numbers which are divisible by 7.

5. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in seventh year. Assuming that the production increases uniformly by fixed number every year.

Find:

- i. The production in the first year
- ii. The production in the 10th year
- iii. The total production in the first seven years.

Solution: Since the production increases uniformly by a fixed number every year, the number of TV sets manufactured in 1st, 2nd, 3rd, years will form an AP

Let us denote the number of TV sets manufactured in the n^{th} year by a_n

$$\text{Then, } a_3 = 600$$

$$\text{Or, } a + 2d = 600 \dots\dots\dots (1)$$

$$\text{And } a_7 = 700$$

$$\Rightarrow a + 6d = 700 \dots\dots\dots 2) \quad (\text{Where } a \text{ is the first term})$$

Subtracting equation 1 from equation 2 we get

$$4d = 100$$

$$\Rightarrow d = \frac{100}{4} = 25$$

Putting $d = 25$ in equation 1 we get

$$a + 2 \times 25 = 600$$

$$a = 600 - 50$$

$$a = 550$$

∴ The production of TV sets in the first year is 550.

ii) Now,

$$a_n = a + 9d$$

$$a_{10} = 550 + 9 \times 25$$

$$= 550 + 225$$

$$= 775$$

∴ production of TV sets in the 10th year is 775.

iii) Also,

$$\Rightarrow S_n = \frac{7}{2} \{2 \times 550 + (7-1) \times 25\} \quad [\because S_n = \frac{n}{2} (2a_1 + (n-1)d)]$$

$$= \frac{7}{2} (1100 + 150)$$

$$= \frac{7}{2} \times 1250$$

$$= 7 \times 625$$

$$= 4375.$$

Therefore, the total production of TV sets in the first 7 years is 4375.

6. How many multiple of 4 lies between 10 and 250?

Solution: First multiple of 4 between 10 and 250 is 12

The last multiple of 4 between 10 and 250 is 248

Therefore the AP is 12, 16, 20, 20,..... 248

Here $a_1=12$, common difference, $d = 4$

And, $a_n = 248$, where n is the number of terms of the above AP

$$a_n = 248$$

$$\Rightarrow a_1 + (n-1)d = 248$$

$$\Rightarrow 12 + (n-1)4 = 248$$

$$\Rightarrow (n-1)4 = 248 - 12$$

$$\Rightarrow n-1 = \frac{236}{4}$$

$$\Rightarrow n-1 = 59$$

$$\Rightarrow n = 59 + 1$$

$$\Rightarrow n = 60$$

Thus, there are 60 multiples of 4 between 10 and 250.

7. Find the sum of the odd numbers, between 0 and 50.

Solution: First odd numbers between 0 and 50 is 1 and the last odd number between 0 and 50 is 49.

Therefore the AP is 1, 3, 5,..... 49

Here $a_1 = 1$, $d = 3 - 1 = 2$

Let n be the number of terms of the AP

$$a_n = 49$$

$$\Rightarrow a_1 + (n-1)d = 49$$

$$\Rightarrow 1 + (n-1)2 = 49$$

$$\Rightarrow (n-1) \times 2 = 49 - 1$$

$$\Rightarrow (n-1) \times 2 = 48$$

$$\Rightarrow n-1 = \frac{48}{2}$$

$$\Rightarrow n-1 = 24$$

$$\Rightarrow n = 24 + 1$$

$$\Rightarrow n = 25$$

Using the formula

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_{25} = \frac{25}{2} (1 + 49)$$

$$= \frac{25}{2} \times 50$$

$$= 25 \times 25$$

$$= 625$$

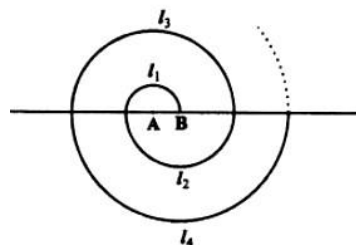
Therefore, the sum of the odd numbers between 0 and 50 is 625.

8. A spiral is made up of successive semi-circles, with centres alternately at A and B, starting with A, of radii 0.5 cm, 1.5 cm, 2.0 cm,.... as shown in the figure 5.4. What is the total length of such spiral made up of thirteen consecutive semi-circles? (Take $\pi = \frac{22}{7}$)

Solution: circumference of 1st semi-circles = 0.5π cm
 circumference of 2nd semi-circles = π cm
 circumference of 3rd semi-circles = 1.5π cm
 circumference of 4th semi-circles = 2π cm

Thus, the AP is $0.5\pi, \pi, 1.5\pi, 2\pi, \dots$

Here $a_1 = 0.5\pi$, $d = \pi - 0.5\pi$



$$=0.5\pi$$

and, $n = 13$

Using the formula

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$\text{Or, } S_n = \frac{n}{2} [a_1 + a_1 + (n-1)d]$$

$$S_{13} = \frac{13}{2} [(2 \times 0.5\pi + (13-1) \times 0.5\pi)]$$

$$S_{13} = \frac{13}{2} (\pi + 12 \times 0.5\pi)$$

$$= \frac{13}{2} (\pi + 6\pi)$$

$$= \frac{13}{2} \times 7\pi$$

$$= \frac{13}{2} \times 7 \times \frac{22}{7} \quad (\because \pi = \frac{22}{7})$$

$$= 13 \times 11$$

$$= 143 \text{ cm}$$

Therefore, the total length of such spiral made up of thirteen consecutive semi-circles is 143 cm.

9. 200 Toys are stacked in the following manner. 20 toys in the bottom row, 19 in the next row, 18 in the row next to it and so on (see fig 5.5) in how many rows are the 200 toys placed and how many toys are in the top row?

Solution: Number of toys in the first row, $a_1 = 20$
Number of toys in the second row, $a_2 = 19$
Number of toys in the third row, $a_3 = 18$

\therefore The sequence becomes 20, 19, 18,....

$$\text{Now, } a_3 - a_2 = 18 - 19 = -1$$

$$a_2 - a_1 = 19 - 20 = -1$$

$$\therefore a_3 - a_2 = a_2 - a_1 = -1$$

Since the difference between any two successive terms is the same everywhere, therefore the given sequence is an AP.

Thus the AP is 20, 19, 18,....

Here $a_1 = 20$, common difference, $d = -1$

Let n be the number of rows

$$\therefore S_n = 200$$

$$\begin{aligned} \Rightarrow \frac{n}{2}(2a_1 + (n-1)d) &= 200 \\ \Rightarrow n\{2 \times 20 + (n-1)(-1)\} &= 200 \times 2 \\ \Rightarrow n(40 - n + 1) &= 400 \\ \Rightarrow n(41 - n) &= 400 \\ \Rightarrow 41n - n^2 &= 400 \end{aligned}$$

Or, $n^2 - 41n + 400 = 0$

$$\begin{aligned} \Rightarrow n^2 - 25n - 16n + 400 &= 0 \\ \Rightarrow n(n-25) - 16(n-25) &= 0 \\ \Rightarrow (n-25)(n-16) &= 0 \end{aligned}$$

Either, $n - 25 = 0$
 $n = 25$

or, $n - 16 = 0$
 $n = 16$

$\therefore n = 16$ or 25

When $n = 16$

$$\begin{aligned} a_{16} &= a_1 + 15d \\ &= 20 + 15 \times (-1) \\ &= 20 - 15 \\ a_{16} &= 5 \end{aligned}$$

And when, $n = 25$

$$\begin{aligned} a_{25} &= a_1 + 24d \\ &= 20 + 24 \times (-1) \\ &= 20 - 24 \\ &= -4 \text{ (is rejected as the number of an AP cannot be negative)} \end{aligned}$$

Therefore, there are 16 number of rows arranged for 200 toys.

And, there are 5 toys in the toy row which is in the 16th row.

10. A sum of Rs 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.

Solution: We know that the formula to calculate simple interest is given by:

$$\text{Simple interest} = \frac{P \times R \times T}{100}$$

So, the interest at the end of the first year = Rs. $\frac{1000 \times 8 \times 1}{100} = \text{Rs. } 80$

The interest at the end of second year = Rs. $\frac{1000 \times 8 \times 2}{100} = \text{Rs. } 160$

The interest at the end of third year = Rs. $\frac{1000 \times 8 \times 3}{100} = \text{Rs. } 240$

Similarly, we can obtain the interest at the end of the 4th year, 4th year, and so on.

So, The interest in rupees at the end of the 1st, 2nd, 3rd, ... years respectively are: 80, 160, 240.....

Here $a_1 = 80, a_2 = 160, a_3 = 240$

Now, $a_3 - a_2 = 240 - 160 = 80$

$$a_2 - a_1 = 160 - 80 = 80$$

Since, the difference between the consecutive terms in the list is 80.

Therefore, the given list of numbers is an AP.

Hence, first term $a_1 = 80$ and common difference, $d = 80$

Therefore, $a_{30} = a_1 + 29d$

$$\begin{aligned} \Rightarrow a_{30} &= 80 + 29 \times 80 \\ &= 80 + 2320 \\ &= 2400 \end{aligned}$$

So, the interest at the end of 30 years will be Rs. 2400.

11. A girl of height 90cm is walking away from the base of a lamp-post at a speed of 1.2m/sec. if the lamp is 3.6 m above the ground; find the length of her shadow after 4 seconds.

Solution:

Let AB denote the lamp-post and CD the girl after walking for 4 seconds away from the lamp-post.

Let DE = x cm be the shadow of the girl.

$BD = 1.2\text{m} \times 4 = 4.8\text{m}$ (\because distance = speed \times time)

$\triangle ABE$ and $\triangle CDE$

$\angle B = \angle D$ (90° each)

$\angle E = \angle E$ (common angles)

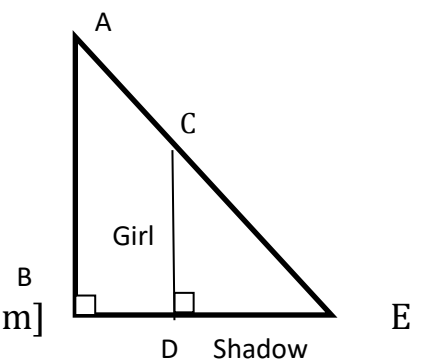
$\therefore \triangle ABC \sim \triangle CDE$ [AA similarity]

$$\frac{DE}{BE} = \frac{CD}{AB} \quad [\text{corresponding sides}]$$

$$\text{or, } \frac{x}{x+4.8} = \frac{0.9}{3.6} \quad [90\text{cm} = \frac{90}{100}\text{m} = 0.9\text{m}]$$

$$\text{or, } \frac{x}{x+4.8} = \frac{9}{36}$$

$$\text{or, } \frac{x}{x+4.8} = \frac{1}{4}$$



or, $4x = x + 4.8$

or, $4x - x = 4.8$

or, $3x = 4.8$

or, $x = \frac{4.8}{3}$

$\therefore x = 1.6\text{m}$

So, the shadow of the girl after walking four seconds is 1.6m

12. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGH$ such that D and H lie on side AB and FE of $\triangle ABC$ and $\triangle FEG$ respectively. If $\triangle ABC \sim \triangle FEG$, Show that

(i) $\frac{CD}{GH} = \frac{AC}{FG}$ (ii) $\triangle DCB \sim \triangle HGE$

Given: $\triangle ABC \sim \triangle FEG$, CD and GH are bisectors of $\angle ACB$ and $\angle EGF$ respectively.

To Prove: (i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) $\triangle DCB \sim \triangle HGE$

Proof: (i) In $\triangle ACD$ and $\triangle FGH$

$\angle A = \angle F$ [$\because \triangle ABC \sim \triangle FEG$]

$\angle ACD = \angle FGH$ [CD bisects $\angle ACB$ and GH bisects $\angle EGF$]

$\therefore \triangle ACD \sim \triangle FGH$ [AA similarity]

$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$ [Corresponding sides of similar Δ]

Proof: (ii) In $\triangle DCB$ and $\triangle HGE$

$\angle B = \angle E$ [$\because \triangle ABC \sim \triangle FGH$]

$\angle DCB = \angle HGE$ [CD and GH are bisectors of $\angle ACB$ and $\angle EGF$]

$\therefore \triangle DCB \sim \triangle HGE$ [AA similarity] Proved.

13. In fig. $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$.

Solution: $DE \parallel OQ$ and $DF \parallel OR$

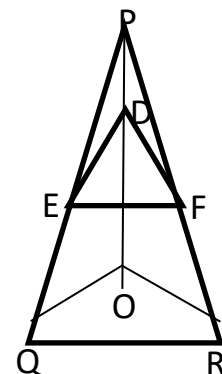
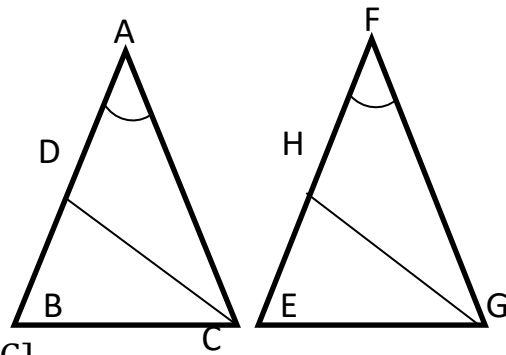
To show: $EF \parallel QR$

Proof: In $\triangle POQ$, $DE \parallel OQ$

By basic proportionality theorem.

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ (1)

In $\triangle POR$, $DF \parallel OR$



$$\therefore \frac{DD}{DO} = \frac{PF}{FR} \quad (2)$$

From equation (1) and (2)

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Now, in ΔPQR , we have E and F are points on PQ and PR respectively such that

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

By converse of proportionality theorem

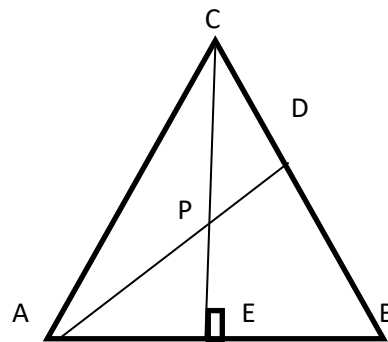
$\therefore EF \parallel QR$. Showed.

14. In fig. altitudes AD and CE of ΔABC intersect each other at the point P show that:

(i) $\Delta AEP \sim \Delta CDP$ (ii) $\Delta ABD \sim \Delta CBE$ (iii) $\Delta AEP \sim \Delta ADB$

Given: ΔABC in which, altitudes AD and CE intersect each other at the point P.

To Prove: (i) $\Delta AEP \sim \Delta CDP$
(ii) $\Delta ABD \sim \Delta CBE$
(iii) $\Delta AEP \sim \Delta ADB$



Proof: (i) In ΔAEP and ΔCDP

$$\angle AEP = \angle CDP [90^\circ \text{ each}]$$

$$\angle APE = \angle CPD [\text{Vertically opposite angles}]$$

$$\therefore \Delta AEP \sim \Delta CDP [\text{AA similarity}]$$

Proof: (ii) In ΔABD and ΔCBE

$$\angle ABD = \angle CBE [\text{Common angles}]$$

$$\angle ADB = \angle CEB [90^\circ \text{ each}]$$

$$\therefore \Delta ABD \sim \Delta CBE [\text{AA similarity}]$$

Proof: (iii) In ΔAEP and ΔADB

$$\angle PAE = \angle DAB [\text{Common angles}]$$

$$\angle AEP = \angle ADB [90^\circ \text{ each}]$$

$$\therefore \Delta AEP \sim \Delta ADB [\text{AA similarity}]$$

15. In fig. ABC and AMP are two right triangles, right angle at B and M respectively. Proved that

(i) $\Delta ABC \sim \Delta AMP$ (ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Given: ABC and AMP are two right triangles, right angled at B and M respectively i.e., $\angle ABC = \angle AMP = 90^\circ$

To Prove: (i) $\Delta ABC \sim \Delta AMP$

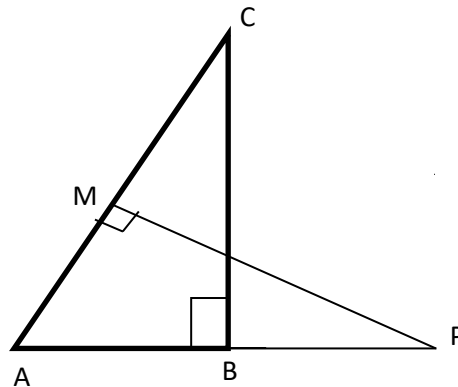
(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Proof: (i) In ΔABC and ΔAMP

$\angle CAB = \angle MAP$ [Common angles]

$\angle ABC = \angle AMP$ [90° each]

$\therefore \Delta ABC \sim \Delta AMP$ [AA Similarity]



Proof: (ii) Since $\Delta ABC \sim \Delta AMP$ [Proved above]

If two triangles are similar, the ratio of their corresponding sides is proportional.

$$\therefore \frac{CA}{PA} = \frac{BC}{MP} = \frac{AB}{AM}$$

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP} \text{ Proved}$$

16. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR . Show that $\Delta ABC \sim \Delta PQR$.

Given: In triangles ABC and PQR, AD and PM are median such that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

To prove: $\Delta ABC \sim \Delta PQR$

Proof: Since $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ [Given]

\therefore D and M are the mid points of BC and QR respectively.

$$\therefore \frac{BC}{PQ} = \frac{2BD}{2QM} = \frac{BD}{QM}$$

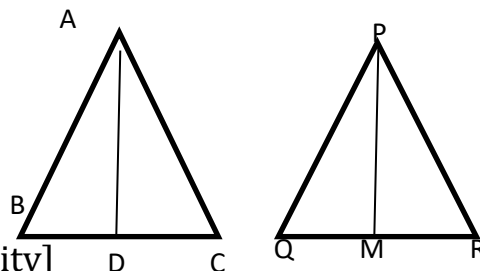
In $\Delta ABD \sim \Delta PQM$

$$\therefore \frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$$

$\Delta ABD \sim \Delta PQM$ [SSS similarity]

$\angle B = \angle Q$ [Corresponding angles]

Now, in ΔABC and ΔPQR



$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$$\angle B = \angle Q \text{ [Proved above]}$$

Hence, $\Delta ABC \sim \Delta PQR$ [Similarity] Proved.

17. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show That ABCD is a trapezium.

Solution: A quadrilateral ABCD whose diagonals AC and BD intersect each other at O such that $\frac{AO}{BO} = \frac{CO}{DO}$.

To prove: Quadrilateral ABCD is a trapezium.

Construction: Draw $EO \parallel BA$, meeting AD at E.

Proof: In ΔABD , $EO \parallel BA$

By basic proportionality theorem

$$\frac{DE}{EA} = \frac{DO}{OB} \quad \text{(i)}$$

But, $\frac{AO}{BO} = \frac{CO}{DO}$ [Given]

$$\Rightarrow \frac{DO}{BO} = \frac{CO}{AO} \quad \text{(ii)}$$

From equation (i) and (ii)

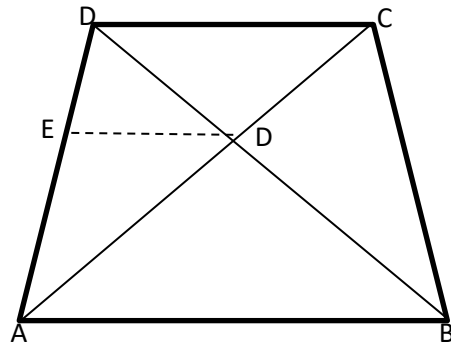
$$\frac{DE}{EA} = \frac{CO}{OA}$$

$$\Rightarrow EO \parallel DC \quad \text{[by converse of B.P.T]}$$

$$\text{But } EO \parallel BA \quad \text{[by construction]}$$

$$\therefore DC \parallel AB$$

Hence, ABCD is a trapezium. Proved.



18. In fig. A, B and C are points on OP, OR and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.

Given: A, B, C are points on OP, OQ and OR respectively such that

$AB \parallel PQ$ and $AC \parallel PR$

To prove: $BC \parallel QR$

Proof: In ΔOPQ , $AB \parallel PQ$

By basic proportionality theorem

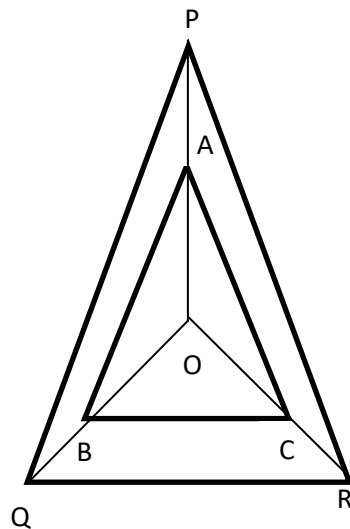
$$\frac{OA}{AP} = \frac{OB}{BQ} \quad \text{(i)}$$

Similarly, in ΔOPR , $AC \parallel PR$

$$\frac{AO}{AP} = \frac{OC}{CR} \quad \text{(ii)}$$

Combining equation (i) and (ii)

$$\frac{OB}{BQ} = \frac{OC}{CR}$$



By converse of basic proportionality theorem.

Line BC divides the ΔOQR in the same ratio

$\therefore BC \parallel QR$ Proved.

19. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$

Given: A parallelogram ABCD, E is the point on the side AD produced and BE intersects CD at F.

To prove: $\Delta ABE \sim \Delta CFB$

Proof: In Parallelogram ABCD

Opposite angles of a parallelogram are equal.

$$\angle A = \angle C \quad (i)$$

Opposite sides of a parallelogram are parallel i.e., $AD \parallel BC$

But AE is AD extended

$\therefore AE \parallel BC$ and BE is the transversal

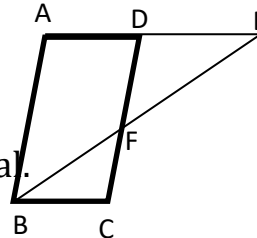
$\therefore \angle AEB = \angle CBF$ (ii) [Alternate interior angles]

In ΔABE and ΔCFB

$$\angle A = \angle C \quad [\text{from eqn (i)}]$$

$$\angle AEB = \angle CBF \quad [\text{from eqn (ii)}]$$

$\therefore \Delta ABC \sim \Delta CFB$ [AA similarity]. Proved.



20. If AD and PM are medians of triangle ABC and POR, respectively where $\Delta ABC \sim \Delta PQR$,

prove $\frac{AB}{PQ} = \frac{AD}{PM}$

Solution:

Given: AD and PM are medians of ΔABC and ΔPOR ,

Also, given $\Delta ABC \sim \Delta PQR$

To prove: $\frac{AB}{PQ} = \frac{AD}{PM}$

Proof: $\therefore \Delta ABC \sim \Delta PQR$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \quad (i) \quad [\text{sides are proportional}]$$

$\therefore AD$ is the median of ΔABC

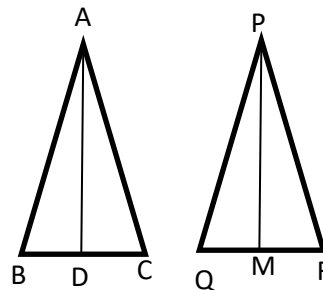
$\Rightarrow D$ is the midpoint of BC

$\Rightarrow BD = CD$

Similarly, $QM = MR$ [$\because M$ is the midpoint of QR]

From equation (i) we have

$$\frac{AB}{PQ} = \frac{BC}{QR}$$



$$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \quad \text{(ii)}$$

Now in ΔABD and ΔPQM

$$\angle B = \angle Q \quad [\because \Delta ABC \sim \Delta PQR]$$

$$\frac{AB}{PQ} = \frac{BD}{QM} \quad [\text{Using (ii)}]$$

$\Rightarrow \Delta ABD \sim \Delta PQM$ (SAS similarity)

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM} \quad [\text{Sides are proportional}]$$

Hence, $\frac{AB}{PQ} = \frac{AD}{PM}$ proved.

21. The angle of elevation of the top of the tower from a point on the ground, which is 30m away from the foot of the tower, is 30° . Find the height of the tower. (use $\sqrt{3} = 1.732$)

Solution:

Let $AB = x$ m be the height of the tower

$BC = 30$ m be a distance of the point C from the foot of the tower

And the angle of elevation is 30°

To find: x

Now, in right ΔABC ,

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{30}$$

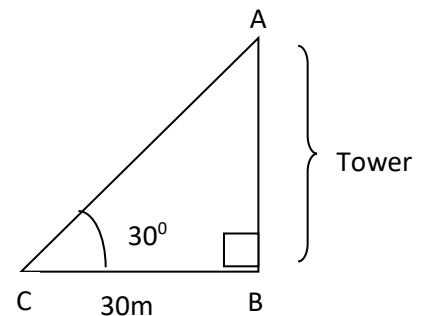
$$\Rightarrow x = \frac{30}{\sqrt{3}}$$

$$\Rightarrow x = \frac{30\sqrt{3}}{3\sqrt{3}}$$

$$\Rightarrow x = 10\sqrt{3} \text{ m}$$

$$\Rightarrow x = 10 \times 1.732$$

$$\Rightarrow x = 17.32 \text{ m}$$



Hence the height of the tower is 17.32m

22. A kite is flying at a height of 60m above the ground. The string attached to the kite is temporarily to a point on the ground. The inclination of the string with the ground is 60° . Find the length of the string, assuming that there is no slack in the string. (use $\sqrt{3} = 1.732$)

Solution:

Let the height of the kite above the ground (AB) = 60m

Let the length of the string (AC) = x m

$\angle ABC = 60^\circ$

In right $\triangle ABC$,

$$\frac{AB}{AC} = \sin 60^\circ$$

$$\Rightarrow \frac{60}{x} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sqrt{3} x = 120$$

$$\Rightarrow x = \frac{120}{\sqrt{3}}$$

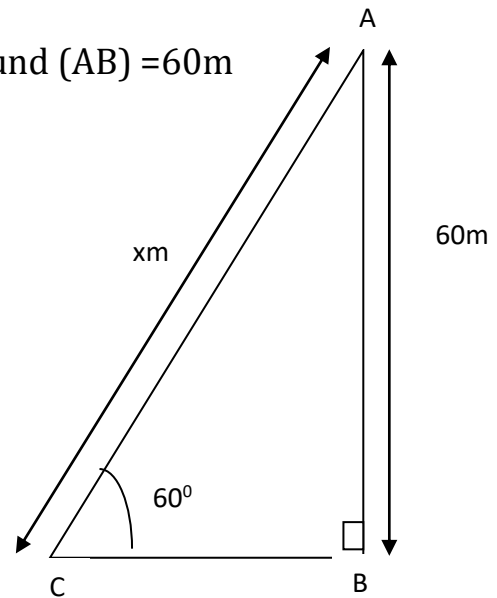
$$\Rightarrow x = \frac{120\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$\Rightarrow x = \frac{120\sqrt{3}}{3}$$

$$\Rightarrow x = 40 \times 1.732$$

$$\Rightarrow x = 69.28 \text{ m}$$

\therefore The length of the string = 69.28 m



23. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8m. Find the height of the tree. (use $\sqrt{3} = 1.732$)

Solution:

Let AC be the height of the tree before it was broken,

And A'B be the broken part of the tree after the storm which makes an angle of 30° with the ground

And A'C = 8m be the distance of the top and foot of the tree.

Now,

In right $\triangle A'BC$,

$$\Rightarrow \tan 30^\circ = \frac{BC}{A'C}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{8}$$

$$\Rightarrow BC = \frac{8}{\sqrt{3}} \quad (i)$$

$$\text{And, } \cos 30^\circ = \frac{A'C}{A'B}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{A'B}$$

$$\Rightarrow A'B = \frac{16}{\sqrt{3}} \quad (ii)$$

$$\text{Now, } AC = A'C \quad (\text{height of the tree})$$

$$= A'B + BC$$

$$= \frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}$$

$$= \frac{16+8}{\sqrt{3}}$$

$$= \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

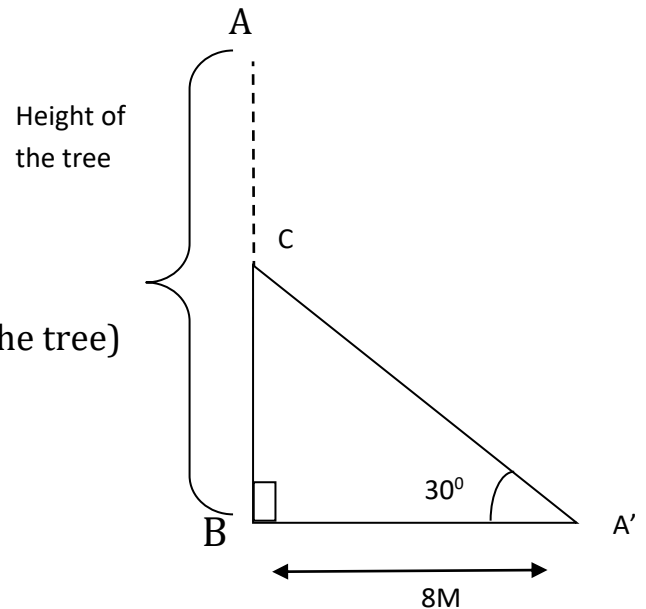
$$= \frac{24\sqrt{3}}{3}$$

$$= 8\sqrt{3} \text{ m}$$

$$AC = 8 \times 1.732 \text{ m}$$

$$AC = 13.856 \text{ m}$$

Hence, the height of the tree is 13.856m



24. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° . If the tower is 50m high, find the height of the building.

Solution:

Let AB be the height of the building

And CD=50m be the height of the tower

The angle of elevation of the top of the building to the foot of the tower is 30°

And the angle of elevation of the top of the tower to the foot of the building is 60°

Now,

In right $\triangle CDB$,

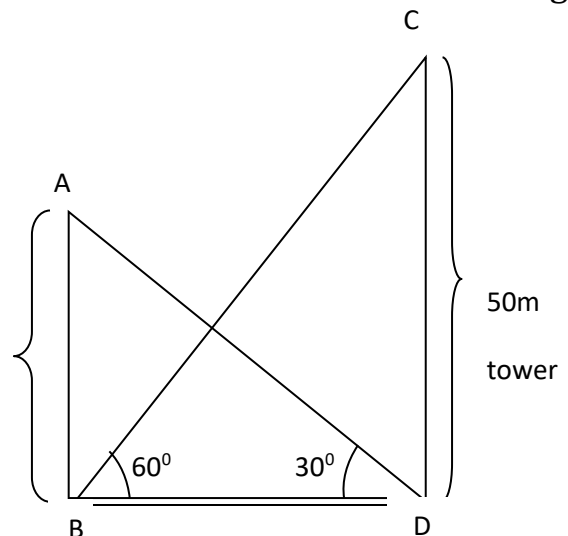
$$\tan 60^\circ = \frac{CD}{BD}$$

$$\Rightarrow \sqrt{3} = \frac{50}{BD}$$

$$\Rightarrow BD = \frac{50}{\sqrt{3}} \quad (i)$$

Also, right $\triangle ABD$

$$\tan 30^\circ = \frac{AB}{BD}$$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{\frac{50}{\sqrt{3}}}$$

$$\Rightarrow AB = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$$

$$= \frac{50}{3}$$

$$= 16.67 \text{ m}$$

Hence the height of the building is 16.67m.

25. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20m high building are 45° and 60° respectively. Find the height of the tower.

Solution: Let AB be the height of the transmission tower and BC = 20m be the height of the building.

A

A

The angle of elevation from a point D on the ground to the top and bottom of the tower are 60° and 45° .

Now, In right $\triangle BCD$

$$\tan 45^\circ = \frac{BC}{CD}$$

$$\Rightarrow 1 = \frac{20}{CD}$$

$$\Rightarrow CD = 20\text{m} \quad (i)$$

Also, in the right $\triangle ACD$

$$\tan 60^\circ = \frac{AC}{CD}$$

$$\Rightarrow \sqrt{3} = \frac{AB+BC}{20} \quad [\text{From (i)}]$$

$$\Rightarrow 20\sqrt{3} = AB+20$$

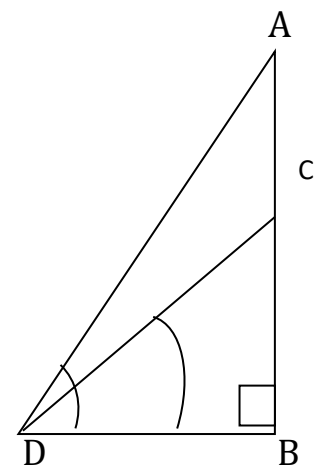
$$\Rightarrow AB = 20\sqrt{3} - 20$$

$$\Rightarrow AB = 20(\sqrt{3} - 1)$$

$$\Rightarrow AB = 20(1.732 - 1)$$

$$\Rightarrow AB = 20 \times 0.732$$

$$\Rightarrow AB = 14.64\text{m}$$



Height of the transmission tower is 14.64m.

26. Two poles of equal heights are standing opposite to each other on either side of the road, which is 30m wide, from a point between them on the road, the angle of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distance of the point from the poles. (use $\sqrt{3} = 1.732$)

Solution: Let AB and CD be the poles of equal height h m.

Let E be the point between the poles.

Let distance between the two poles: $BC=80\text{m}$

Where $BE = x\text{m}$ and $EC= (80 - x)\text{m}$

Let $\angle AEB = 60^\circ$ be the angle of elevation of pole AB from the point E on the road.

Let $\angle CED = 30^\circ$ be the angle of elevation of pole CD from point E on the ground.

Now, In right $\triangle ABC$

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = x\sqrt{3} \quad (i)$$

and, right $\triangle CED$

$$\tan 30^\circ = \frac{CD}{EC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{80-x}$$

$$\Rightarrow h\sqrt{3} = 80 - x$$

$$\Rightarrow x\sqrt{3} \times \sqrt{x} = 80 - x$$

$$\Rightarrow 3x = 80 - x$$

$$\Rightarrow 3x + x = 80$$

$$\Rightarrow 4x = 80$$

$$\Rightarrow x = \frac{80}{4}$$

$$\Rightarrow x = 20\text{m}$$

Putting $x = 20\text{m}$ in equation (i) we get

$$h = 20\sqrt{3}$$

$$\Rightarrow h = 20 \times 1.732$$

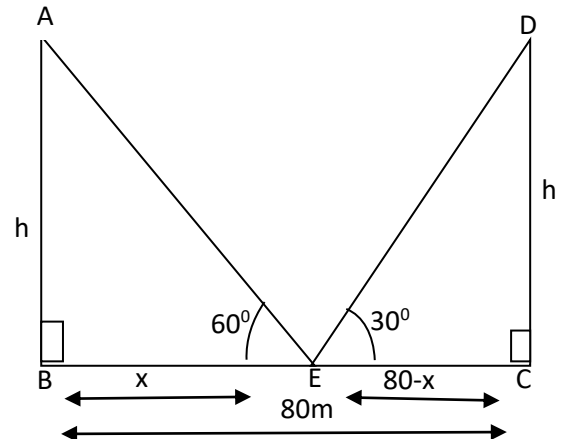
$$\Rightarrow h = 34.64\text{m}$$

$$\begin{aligned} \text{And } 80 - x &= 80 - 20 \\ &= 60\text{m} \end{aligned}$$

Hence the height of the equal poles is 34.64m

Distance of point E from the pole AB is 20m

And, distance of point E from the pole CD 60m



27. The shadow of a tower standing on a level ground is found to be 40m longer when the sun's altitude is 30° than when it is 60° . Find the height of the tower.

Solution: Let AB be the tower and BC is the length of the shadow

When the sun's altitude is 60°

Let DB be the length of the shadow when the sun's altitude is

30°

Now, Let AB be hm and BC be xm

According to question

DB is 40cm longer than BC

So, $DB = (40 + x)$

Now, in right $\triangle ABC$

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = \sqrt{3}x \quad (i)$$

Now, in right $\triangle ABD$

$$\tan 30^\circ = \frac{AB}{DB}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+40}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{x+40} \quad \{\text{using (i)}\}$$

$$\Rightarrow x+40 = 3x \quad \{\sqrt{3} \times \sqrt{3} = 3\}$$

$$\Rightarrow 3x - x = 40$$

$$\Rightarrow 2x = 40$$

$$\Rightarrow x = \frac{40}{2} = 20$$

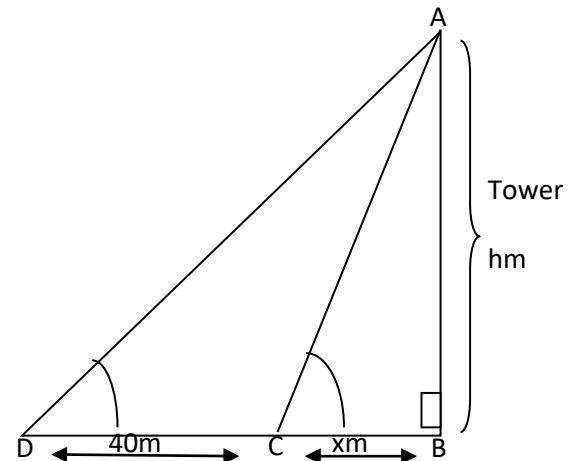
Putting $x = 20$ in equation (i) we get

$$\Rightarrow h = 20\sqrt{3}$$

$$\Rightarrow h = 20 \times 1.732$$

$$\Rightarrow h = 34.64\text{m}$$

Hence, height of the tower is 34.64m.



28. From a window 15m high above the ground, in a street the angle elevation and depression of the top and foot of another house on opposite side of the street one 30° and 45° respectively. Show that the height of the opposite house is 23.66m
(use $\sqrt{3} = 1.732$)

Solution: Let the height of opposite house AB = hm

Let the height of another house DC = 15m

Let the distance between two houses

$$BC = xm$$

$$\therefore BC = EB = xm$$

$$DC = EB = 15m$$

$$AE = AB - EB = (h - 15)m$$

$$\angle ADE = 30^\circ$$

$$\angle EDB = \angle DBC = 45^\circ$$

In right $\triangle DCB$,

$$\frac{DC}{BC} = \tan 45^\circ$$

$$\Rightarrow \frac{15}{x} = 1$$

$$\Rightarrow x = 15 \quad (i)$$

Again, in right $\triangle AED$

$$\frac{AE}{ED} = \tan 30^\circ$$

$$\Rightarrow \frac{h-15}{15} = \frac{1}{\sqrt{3}} \text{ {Using equation (i)}}$$

$$\Rightarrow h - 15 = \frac{15}{\sqrt{3}}$$

$$\Rightarrow h - 15 = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow h - 15 = \frac{15 \times 1.732}{3}$$

$$\Rightarrow h - 15 = 8.66$$

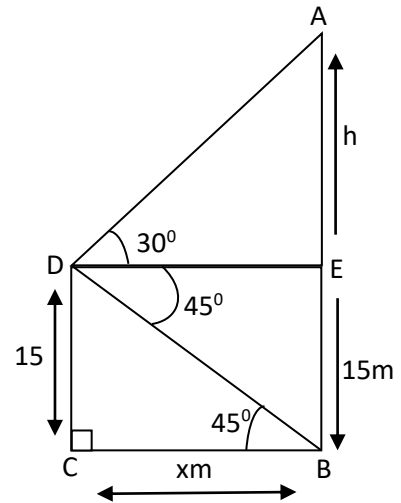
$$\Rightarrow h = 8.66 + 15$$

$$\Rightarrow h = 23.66$$

\therefore Height of opposite house

$$= AB$$

$$= 23.66m$$



29. A straight highway leads to the foot of a tower of height 50m. From the top of the tower, the angles of depression of two cars standing on the highway are 30° and 60° respectively. What is the distance between two cars and how far is each car from the tower?

Solution: Let the height of tower, AB = 50m

Let the distance of 1st position of car from the tower, C = xm

Let the distance of 2nd position of car from the tower, D = ym

∴ Distance, between two cars:

$$CD = BC - BD = (x - y)m$$

$$\angle PAC = \angle ACB = 30^\circ$$

$$\angle PAD = \angle ADB = 60^\circ$$

In right $\triangle ABC$

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\Rightarrow \frac{50}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = 50\sqrt{3}$$

$$\Rightarrow x = 50 \times 1.732$$

$$\Rightarrow x = 86.60$$

Again, In right $\triangle ABD$,

$$\frac{AB}{BD} = \tan 60^\circ$$

$$\Rightarrow \frac{50}{y} = \sqrt{3}$$

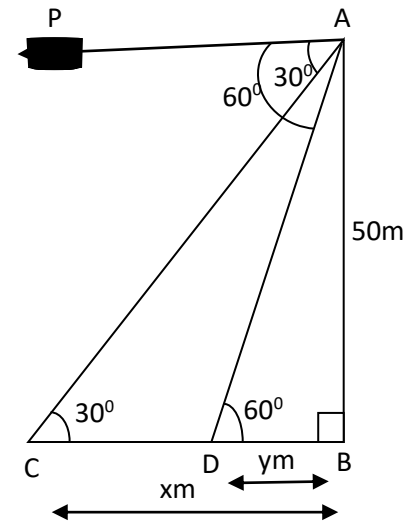
$$\Rightarrow \sqrt{3}y = 50$$

$$\Rightarrow y = \frac{50}{\sqrt{3}}$$

$$\Rightarrow y = \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow y = \frac{50 \times 1.732}{3}$$

$$\Rightarrow y = \frac{86.60}{3} = 28.87m$$



∴ Distance between the two cars:

$$= x - y$$

$$= 86.60 - 28.87$$

$$= 57.73m$$

Hence, Distance from the tower of 1st car, $x = 86.60m$

Distance from the tower of the 2nd car, $y = 28.87m$

30. A 1.5m Tall boy is standing at some distance from a 30m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building. ($\sqrt{3}=1.73$)

Solution: Let $AB = GF = EC = 1.5m$ be the height of the boy

Let $CD = 30\text{m}$ be the height of the building

$$\begin{aligned}\therefore DE &= CD - EC \\ &= 30\text{m} - 1.5\text{m} \\ &= 28.5\text{m}\end{aligned}$$

$$\angle DAE = 30^\circ$$

$$\angle DGE = 60^\circ$$

$$AG = AE - GE$$

In right $\triangle DEA$

$$\frac{DE}{AE} = \tan 30^\circ$$

$$\Rightarrow \frac{DE}{AE} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{28.5}{AE} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AE = 28.5 \times \sqrt{3} \quad (i)$$

Again, In right $\triangle DEG$

$$\frac{DE}{GE} = \tan 60^\circ$$

$$\Rightarrow \frac{DE}{GE} = \sqrt{3}$$

$$\Rightarrow \frac{28.5}{GE} = \sqrt{3}$$

$$\Rightarrow GE = \frac{28.5}{\sqrt{3}}$$

$$\Rightarrow GE = \frac{28.5 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{28.5 \times \sqrt{3}}{3} \quad (ii)$$

$$\text{Now, } AG = 28.5 \times \sqrt{3} - \frac{28.5 \times \sqrt{3}}{3}$$

$$= 28.5\sqrt{3} \left(1 - \frac{1}{3}\right)$$

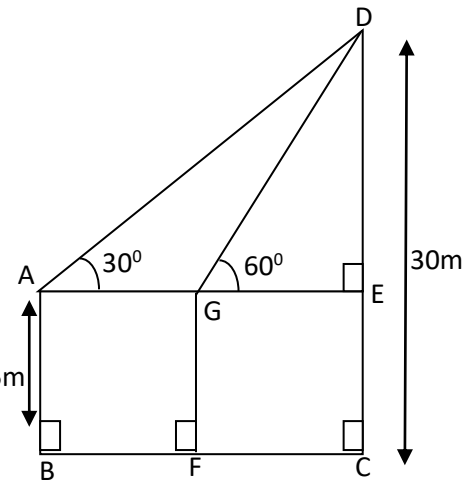
$$= 28.5\sqrt{3} \left(\frac{3-1}{3}\right)$$

$$= \frac{28.5 \times 1.732 \times 2}{3} \text{ m}$$

$$= \frac{98.61}{3} \text{ m}$$

$$= 32.87 \text{ m}$$

\therefore The required distance he walked towards the building is 32.87



31. A Toy is in the form of a cone mounted on a hemisphere. The diameter of the base of a cone is 18cm and its height is 12cm. calculated the total surface area of a toy. (take $\pi = 3.14$)

Solution:

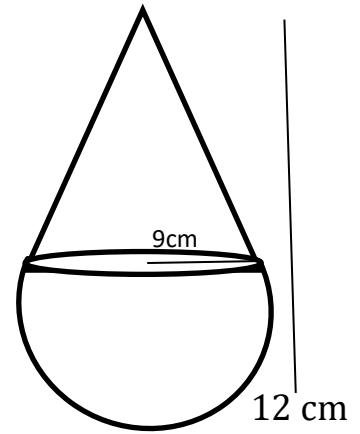
$$\text{Radius of the hemisphere, } r = \frac{18}{2} = 9\text{cm}$$

$$\text{Curved surface area of the hemisphere} = 2 \pi r^2$$

$$= 2 \times 3.14 \times 9 \times 9$$

$$= 508.68 \text{ cm}^2$$

Radius of the cone, $r = 9\text{cm}$
 Height of the cone, $h = 12\text{cm}$
 Slant height of the cone, $l = \sqrt{r^2 + h^2}$
 $= \sqrt{9^2 + 12^2}$
 $= \sqrt{81 + 144}$
 $= \sqrt{225}$
 $= 15$
 $\therefore l = 15\text{cm}$



Curved surface area of the cone $= \pi r l$
 $= 3.14 \times 9 \times 15$
 $= 423.90 \text{ cm}^2$

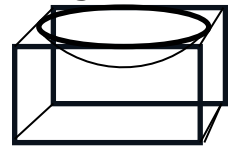
\therefore The total surface area of the toy = curved surface area of the cone + curved surface area of the hemisphere
 $= 508.68 + 423.90$
 $= 932.58 \text{ cm}^2$

32. A hemisphere depression is cut out from one face of a cubical wooden block such that the diameter of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Solution:

Edge of the cube $= l$

\therefore Radius of the hemisphere, $r = \frac{l}{2}$



Now,

Surface area of the remaining solid
 $=$ TSA of the cube $-$ area of the top of the hemispherical part $+ \text{CSA of the hemisphere}$

$$= 6l^2 - \frac{\pi l^2}{4} + \frac{2\pi l^2}{4}$$

$$= \frac{24l^2 - \pi l^2 + 2\pi l^2}{4}$$

$$= \frac{24l^2 + \pi l^2}{4}$$

$$= \frac{l^2(24 + \pi)}{4}$$

$\therefore \text{Area of a circle} = \pi r^2 \text{ and CSA of hemisphere} = 2\pi r^2$

Hence, Surface area of the remaining solid is $\frac{l^2}{4}(24 + \pi)$

33. A Gulabjamun, contain sugar syrup up to 30% of its volume. Find approximately how much syrup would he found in 45 gulabjamun shaped like a cylinder with two hemispherical edges, with length 5cm and diameter 2.8cm.

Solution:

$$\text{Radius of the gulabjamun, } r = \frac{2.8}{2} = 1.4 \text{ cm}$$

$$\begin{aligned} \text{Height of the cylindrical portion, } h &= 5 - (2 \times 1.4) \\ &= 2.2 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Volume of the cylindrical portion} &= \pi r^2 h \\ &= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 \text{ cm}^3 \\ &= 13.552 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume of two hemispherical ends} &= 2 \times \frac{2}{3} \pi r^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4 \\ &= \frac{34.496}{3} \\ &= 11.50 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Total volume of gulabjamun} &= (13.552 + 11.50) \text{ cm}^3 \\ &= 25.052 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Amount of sugar syrup in one gulabjamun} &= 30\% \text{ of } 25.052 \\ &= 7.52 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \therefore \text{Amount of sugar syrup in 45 gulabjamun} &= 45 \times 7.52 \text{ cm}^3 \\ &= 338.20 \text{ cm}^3 \\ &= 338 \text{ cm}^3 \text{ (approx)} \end{aligned}$$

34. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depression is 0.5 cm and depth is 1.4 cm. Find the volume of wood in the entire stand. (Textbook Q4 ex 12.2) (use $\pi = 22/7$)

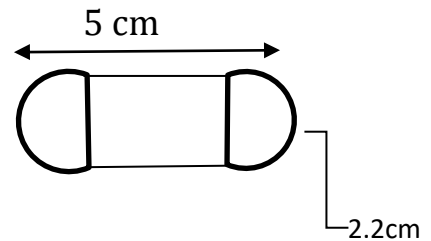
Solution:

$$\begin{aligned} \text{Volume of pen stand (cuboid)} &= 15 \times 10 \times 3.5 \\ &= 525 \text{ cm}^3 \end{aligned}$$

$$\text{Radius of conical depression, } r = 0.5 = \frac{1}{2} \text{ cm}$$

$$\text{Height of conical depression, } h = 1.4 \text{ cm}$$

$$\begin{aligned} \text{Volume of conical depression} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times 1.4 \\ &= \frac{2.2}{6} \\ &= \frac{11}{30} \text{ cm}^3 \end{aligned}$$



$$\therefore \text{Volume of 4 conical depression} = 4 \times \frac{11}{30} \times \text{cm}^3$$

$$= \frac{22}{15} \text{ cm}^2 \text{ or } 1.47$$

\therefore Volume of cuboid in the stand = Volume of pen stand - volume of 4 conical depression

$$= 525 - 1.47$$

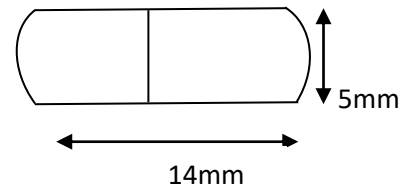
$$= 523.53 \text{ cm}^3$$

35. A mediocre capsule is in the shaped of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14mm and the diameter of the capsule is 5mm. find its surface area.

Solution:

Total length of capsule = 14mm

Diameter of capsule = 5 mm



Then, radius of each hemispherical end, $r = \frac{5}{2} \text{ mm}$

\therefore Surface area of the two hemispherical ends, $A_1 = 2 \times 2 \pi r^2$

$$= 4 \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \text{ mm}^2$$

$$= \frac{550}{7} \text{ mm}^2$$

Again, length of the cylindrical portion of the capsule, $h = \text{Total length of capsule} - 2 \times \text{hemispherical ends}$.

$$= 14 \text{ mm} - 2 \times \frac{5}{2} \text{ mm}$$

$$= 9 \text{ mm}$$

\therefore Surface area of the cylindrical portion, $A_2 = 2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{5}{2} \times 9 \text{ mm}^2$$

$$= \frac{990}{7} \text{ mm}^2$$

Hence, the required total surface area of the mediocre capsule = $A_1 + A_2$

$$= \frac{550}{7} + \frac{990}{7} \text{ mm}^2$$

$$= \frac{1540}{7} \text{ mm}^2$$

$$= 220 \text{ mm}^2$$

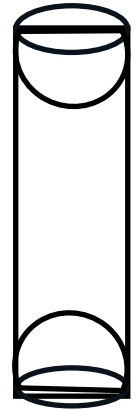
36. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure 12.11. If the height of the cylinder is 10cm and its base is of radius 3.5cm, find the total surface area of the article.

Solution:

$$\begin{aligned} \text{Radius of the cylinder, } r &= 3.5\text{cm} \\ \text{Height of the cylinder, } h &= 10\text{cm} \end{aligned} \quad \left\{ \because r = \frac{1}{2} \times 7 = 3.5 \right\}$$

$$\begin{aligned} \text{CSA of the cylinder} &= 2\pi rh \\ &= 2 \times \frac{22}{7} \times 3.5 \times 10 \text{ cm}^2 \\ &= 220 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{CSA of the scooped hemisphere} &= 2\pi r^2 \\ &= 2 \times \frac{22}{7} \times 3.5 \times 3.5 \text{ cm}^2 \\ &= 77 \text{ cm}^2 \end{aligned}$$



∴ Required surface area of the article

$$\begin{aligned} &= \text{CSA of cylinder} + 2(\text{CSA of one scooped hemisphere}) \\ &= 220 \text{ cm}^2 + 2 \times 77 \text{ cm}^2 \\ &= 220 \text{ cm}^2 + 154 \text{ cm}^2 \\ &= 374 \text{ cm}^2 \end{aligned}$$

37. From a solid cylinder whose height is 2.4cm and diameter 1.4cm, a conical cavity of the same height and same diameter is hallowed out. Find the total surface area of the remaining solid the nearest cm^2 .

Solution: Radius of the base of the cylinder, $r = \frac{1.4}{2} = 0.7 \text{ cm}$

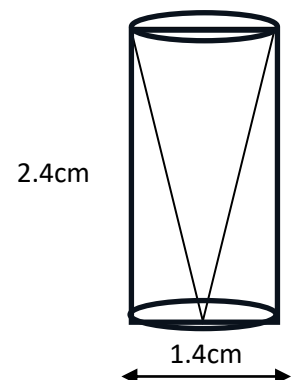
Height of the cylinder, $h = 2.4 \text{ cm}$

$$\begin{aligned} \text{CSA of the cylinder} &= 2\pi rh \\ &= 2 \times \frac{22}{7} \times 0.7 \times 2.4 \text{ cm}^2 \\ &= 10.56 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Slant height of the cone, } l &= \sqrt{r^2 + h^2} \\ &= \sqrt{0.7^2 + 2.4^2} \\ &= \sqrt{0.49 + 5.76} \\ &= \sqrt{6.25} \\ &= 2.5 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{CSA of the cone} &= \pi rl \\ &= \frac{22}{7} \times 0.7 \times 2.5 \\ &= 5.5 \text{ cm}^2 \end{aligned}$$

Area of the base of a cone = πr^2



$$= \frac{22}{7} \times 0.7 \times 0.7 \text{ cm}^2$$

$$= 1.54 \text{ cm}^2$$

$$\text{TSA of the remaining solid} = (10.56 + 5.5 + 1.54)$$

$$= 17.6 \text{ cm}^2$$

$$= 18 \text{ cm}^2 \text{ (to the nearest cm}^2\text{)}$$

38. Two cubes each of volume 64 cm^3 are joined end to end. Find the surface of the resulting cuboid.

Solution:

$$\text{Volume of a cube} = 64 \text{ cm}^3$$

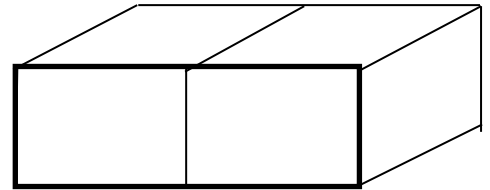
Let the edge of a cube be $x \text{ cm}$

$$\therefore \text{the volume of the cube} = x^3 \text{ cm}^3$$

$$\Rightarrow x^3 = 64$$

$$\Rightarrow x^3 = 4^3$$

$$\Rightarrow x = 4$$



When two cubes are joined together,

$$\text{Length of the new cuboid, } l = 2x$$

$$= 2 \times 4$$

$$= 8 \text{ cm}$$

$$\text{Width of the new cuboid, } b = x \text{ cm} = 4 \text{ cm}$$

$$\text{Height of the new cuboid, } h = x \text{ cm} = 4 \text{ cm}$$

$$\therefore \text{Surface area of the resulting cuboid} = 2(lb + bh + lh)$$

$$= 2(8 \times 4 + 4 \times 4 + 8 \times 4)$$

$$= 2(32 + 16 + 32)$$

$$= 2 \times 80$$

$$= 160 \text{ cm}^2$$

39. Rachel an engineering, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 30 cm and its length is 12 cm . if each one has height of 2 cm , find the volume of air contained in the model that Rachel made (assume the outer and inner dimensions of the model to be nearly the same).

Solution:

Radius of the cone = Radius of the cylinder = r

$$\begin{aligned} \text{Hence, } r &= \frac{1}{2} \times \text{diameter} \\ &= \frac{1}{2} \times 3 \text{ cm} \\ &= \frac{3}{2} \text{ cm} \end{aligned}$$

Total height of the model = 12cm

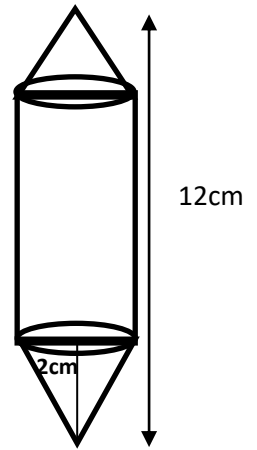
Height of the cone, $h_2 = 2 \text{ cm}$

$$\begin{aligned} \therefore \text{Height of the cylinder, } h_1 &= 12 - 4 \\ &= 8 \text{ cm} \end{aligned}$$

Hence, volume of the model = 2 × volume of cone + Volume of cylinder

$$\begin{aligned} &= 2 \times \frac{1}{3} \pi r^2 h_2 + \pi r^2 h_1 \\ &= \frac{\pi r^2}{3} (2h_2 + 3h_1) \\ &= \frac{22 \times 3 \times 3}{7 \times 3 \times 2 \times 2} (2 \times 2 + 3 \times 8) \\ &= \frac{22 \times 33}{7 \times 2 \times 2} (4 + 24) \\ &= \frac{11 \times 3}{7 \times 2} \times 28 \\ &= 66 \text{ cm}^3 \end{aligned}$$

∴ The volume of air contained in the model is 66 cm³.



40. A vessel is in the form of an inverted cone. Its height is 8cm and the radius of its top, which is open, is 5cm. It is filled with water up to the brim when lead shots, each of which is a sphere of radius 0.5cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Solution:

Radius of conical vessel, $R = 5 \text{ cm}$

Height of conical vessel, $h = 8 \text{ cm}$

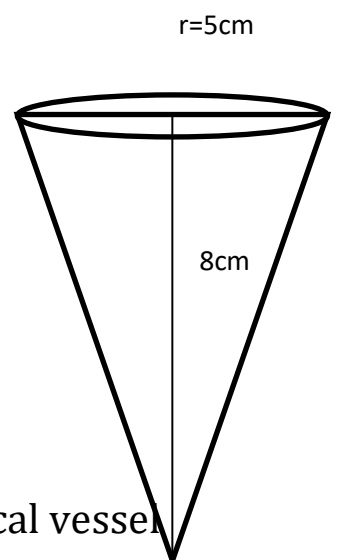
Radius of each lead shot, $r = 0.5 \text{ cm}$

$$= \frac{1}{2} \text{ cm}$$

Let the number of lead shots be 'n'.

Therefore,

$$\text{Volume of } n \text{ lead shots} = \frac{1}{4} \text{ volume of conical vessel}$$



$$\Rightarrow n \times \frac{4}{3} \times \pi r^3 = \frac{1}{4} \times \frac{1}{3} \times \pi R^2 h$$

$$\Rightarrow n = \frac{3 \times \pi \times h \times R^2}{4 \times 4 \times 3 \times \pi \times r^3}$$

$$\Rightarrow n = \frac{5 \times 5 \times 8}{4 \times 4 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}}$$

$$\Rightarrow n = 100$$

\therefore The number of lead shots dropped in the vessel is 100.

Sample Question Paper
(SSLC Examination 2024-25)

Mathematics

(New Course - NCERT Textbook)

by

Meghalaya Board of School Education (MBOSE)

A. The Scheme of Examination

	Maximum Marks	Pass Marks
Theory Examination	80	24
Internal Assessment	20	6
Total	100	30

B. Scheme of Theory Examination

Section	Type of Questions	Marks for Each Question	No. of questions to be attempted/ no. of questions given	Total Marks
Section-A	MCQs	1	30/30	1x30=30
Section-B	Very Short Answer Questions	2	6/9	2x6=12
Section-C	Short Answer Questions	3	6/9	3x6=18
Section-D	Long Answer Questions	5	4/7	5x4=20
Total Marks				80

C. Scheme of Internal Assessment

The Internal Assessment can be done through anyone of the following:

1. Project Work
2. Written Tests
3. Assignments (Class work or Home Work)

D. Content Weightage in Theory Examination

The chapter-wise weightage shown below is only indicative for the purpose of information of teachers while prioritising different chapters during teaching or assessment. Though the weightage in Theory Examination conducted by MBOSE would broadly follow the following pattern, there may still be some variation.

Subject	Chapter	Marks
Algebra	<ul style="list-style-type: none">• Real Numbers• Polynomials• Pairs of Linear Equations in Two Variables• Quadratic Equations• Arithmetic Progression	27
Coordinate Geometry	<ul style="list-style-type: none">• Coordinate Geometry	05
Trigonometry	<ul style="list-style-type: none">• Introduction to Trigonometry• Some Applications of Trigonometry	09
Geometry and Mensuration	<ul style="list-style-type: none">• Triangles• Circles• Arrears related to Circles• Surface Areas and Volumes	29
Statistics & Probability	<ul style="list-style-type: none">• Statistics• Probability	10
Total		80

Sample Question Paper

Mathematics
Class-X

Question Paper Code: XY

Time: 3 hours

Max Marks: 80 (Pass Marks: 24)

General Instructions:

1. Please check that this Question Paper contains 55 Questions.
2. Question Paper Code given above should be written on the Answer Book, in the space provided, by the Candidate.
3. 15 minutes time is given for the candidates to read the Question paper. The Question Paper will be distributed 15 minutes before the scheduled time of the examination. In these 15 minutes, the candidates should only read the instructions and questions carefully and should not write answers on the Answer Sheet.
4. The Question Paper contains 4 sections, Section A, B, C and D.
5. Section-A contains Multiple Choice Questions (MCQ). Choose the most appropriate answer from the given options. The answers to this Section must be provided in the boxes provided in the Answer Sheet. Answers provided anywhere else will not be counted for marking.
6. Section-B contains Very Short Answer Questions. Answer the questions briefly, in minimum 3 steps.
7. Section-C contains Short Answer Questions. Answer the questions in minimum 5 steps.
8. Section-D contains Long Answer Questions. Answer the questions in minimum 8 steps.
9. Use of calculators/ mobile phone/ any electronic device is NOT ALLOWED.

Section- A

Multiple Choice Questions: Attempt **ALL** Questions. (30 X 1 = 30 marks)

- The product of a non-zero rational number and an irrational number is :
(A) An irrational number (B) a rational number (C) one (D) zero
- If the product of two numbers is 540 and their LCM is 30, then their HCF is:
(A) 15 (B) 16 (C) 18 (D) 24
- Which of the following is a quadratic polynomial?
(A) $x + 7$ (B) $x^2 - 2$ (C) $x^3 + 4x + 9$ (D) $x^4 + 3x^3 + 2x + 7$
- A polynomial of degree 3 is called a:
(A) Linear polynomial (B) quadratic polynomial
(C) cubic polynomial (D) biquadratic polynomial
- The pair of equations $x = a$ and $y = b$ graphically represents lines which are:
(A) parallel (B) coincident (C) intersecting at (a, b) (D) intersecting at (b, a)
- The system of equations $-3x + 4y = 5$ and $\frac{9}{2}x - 6y + \frac{15}{2} = 0$ has :
(A) Unique solution (B) infinite many solutions (C) no solutions (D) none of these
- The sum of the roots of the equation $x^2 - 6x + 5 = 0$ is:
(A) 5 (B) - 5 (C) 6 (D) -6
- The product of the roots of the equation $x^2 - 6x + 5 = 0$ is:
(A) 5 (B) - 5 (C) 6 (D) -6
- The 9th term of an AP: 3, 8, 13, 18, is:
(A) 43 (B) 23 (C) 93 (D) 113
- The first three terms of an AP when the first term, $a = 4$ and common difference, $d = 6$ are:
(A) 4, - 2, - 8 (B) 4, 10, 16 (C) 10, 16, 22 (D) -10, -16, -22

11. All geometrical congruent figures are:

- (A) Not similar (B) similar (C) unequal (D) none of the above

12. The ratio of any two corresponding sides in two equiangular triangles is always:

- (A) Complementary (B) different (C) equal (D) none of the above

13. If two angles of one triangle are respectively equal to two angles of another triangle then the two Triangles are similar. This is referred to as the:

- (A) AA Similarity Criterion for two triangles
(B) SAS Similarity Criterion for two triangles
(C) AAA Similarity Criterion for two triangles
(D) SSS Similarity Criterion for two triangles

14. The distance of a point P (3, 4) from origin is:

- (A) 1 unit (B) 3 units (C) 5 units (D) 7 units

15. The midpoint of the line segment joining the points A (- 2, 8) and B (- 6, - 4) is:

- (A) (4, 2) (B) (- 4, 2) (C) (4, - 2) (D) (- 4, - 2)

16. The value of $1 + \tan^2 45^\circ$ is:

- (A) 0 (B) - 1 (C) 1 (D) 2

17. If $\cos \theta = 1$ then the value of θ is:

- (A) 0° (B) 30° (C) 60° (D) 90°

18. How many tangents can be drawn parallel to the secant of a circle?

- (A) One (B) two (C) three (D) infinitely many

19. If a tangent PQ at a point P of a circle of radius 5cm meets a line through the Centre O such that OQ = 12 cm then the length PQ is :

- (A) 12cm (B) 13 cm (c) 8.5cm (D) $\sqrt{119}$ cm

20. If a tangent PA and PB from a point P to a circle with Centre O are inclined to each other at an angle of 80° , then $\angle POA$ is equal to:

- (A) 50° (B) 60° (C) 70° (D) 80°

21. The angle made by the minute hand of a clock at its Centre in 15 minutes duration is:

- (A) 60° (B) 80° (C) 90° (D) 180°

22. If the circumference of a circle increases from $2\pi r$ to $4\pi r$ then its area is:

- (A) four times (B) tripled (C) doubled (D) halved

23. The total surface area of a hemisphere of radius R units is:

- (A) πR^2 sq. units (B) $2\pi R^2$ sq. units (C) $3\pi R^2$ sq. units (D) $4\pi R^2$ sq. units

24. During conversion of a solid from one shape to another, the volume of the new shape will:

- (A) Increase (B) decreases (C) remain unaltered (D) be doubled

25. If the surface area of a sphere is 616 cm^2 its diameter is:

- (A) 7 cm (B) 14 cm (C) 28 cm (D) 56 cm

26. The middle most observation of every data arranged in order is called:

- (A) Median (B) mode (C) mean (D) deviation

27. A numerical data is said to be multimodal if it has:

- (A) Single mode (B) two modes (C) three modes (D) more than three modes

28. Which of the following cannot be the probability of an event?

- (A) $\frac{2}{3}$ (B) -1.5 (C) 15 % (D) 0.7

29. A Child has a die whose six faces marked with letters A, B, C, D, E, A. When the die is thrown once then the probability of getting A is:

- (A) 2 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

30. The probability that it will rain today is 0.87, then the probability that it will not rain today is:

- (A) 0.13 (B) 1.87 (C) $\frac{87}{100}$ (D) 1.03

Section - B

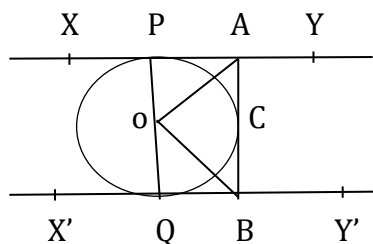
Very Short Answer Questions: Answer **any 6 (six)**. (2x6=12 marks)

31. Express 5005 as a product of its prime factors.
32. Solve the following system of linear equations: $2x + 3y = 5$ and $3x - 4y = -1$
33. Find the length and breadth of a rectangular mango grove whose length is twice its breadth and its area is 800 m^2 ?
34. Prove that $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$
35. If $\cot \theta = 7/8$, evaluate $\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$
36. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.
37. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.
38. Prove that $3 + 2\sqrt{5}$ is an irrational number. It is given that $\sqrt{5}$ is an irrational number.
39. Show that the number $7 \times 11 \times 13 + 13$ is a composite number.

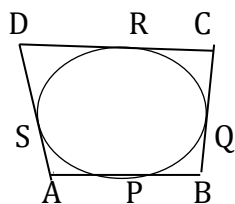
Section - C

Short Answer Questions: Answer **any 6 (six)**. (3x6=18 marks)

40. Find the ratio in which the line segment joining the points A (1, -5) and B (-4, 5) is divided by the x-axis.
41. Find the coordinates of a point A, where AB is the diameter of a circle whose Centre is (2, -3) and B is (1, 4).
42. Rohan's mother is 26 years older than him. The product of their ages 3 years from now will be 360 years. Find their present ages?
43. In the given figure, XY and X'Y' are two parallel tangents to a circle with Centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90^\circ$.



44. A Quadrilateral ABCD is drawn to circumscribe a circle as shown in the adjoining figure. Prove that $AB + CD = AD + BC$



45. In a circle of radius 21 cm, an arc subtends an angle of 60° at the Centre. Find:
 (i) The length of the arc; and
 (ii) area of the sector formed by the arc. (use $\pi = 22/7$)
46. The following table shows the ages of the patients admitted in a hospital during the year. Based on the information, find median of the given data.

Ages (in years)	5-15	15-25	25-35	35-45	45-55	55-65
Number of patients	6	11	21	23	14	5

47. Based on the information given in Question no. 46, find mean of the given data.
48. If α and β are zeroes of the polynomial $P(x) = 3x^2 - 2x - 6$, then find the value of $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

Section - D

Long Answer Questions: Answer **any 4 (four)** (4x5=20 marks)

49. A Tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the Cylindrical part are 2.1 m and 4 m respectively; and the slant height of the top is 2.8 m. Find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹500 per meter. (Use $\pi = 22/7$)
50. If α and β are zeroes of the polynomial $p(x) = ax^2 + bx + c$, then evaluate $(\alpha - \beta)^2$.
51. If the 5th and 12th term of an AP are 30 and 65 respectively, then find the sum of its first 20 terms?
52. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, and then prove that the other two sides are divided in the same ratio.
53. In the give figure PA, QB ad RC are each perpendicular to AC, such that PA =x, QB = z, RC = y, AB= a, and BC = b. prove that $1/x + 1/y = 1/z$.
54. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° . Find the height of the tower and the width of the canal?

55. The mileage (Km per litre) of 50 cars of the same model was tested by a manufacturer and details are tabulated as given below:

Mileage(Km per litre)	10-12	12-14	14-16	16-18
Number of cars	7	12	18	13

Find the mean mileage.

*** End of the Question Paper ***