CM IMPACT Guidebook for Teachers
(With Important Questions and Answers)

Mathematics
Class-X
(Old Course)
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Section-A

(Multiple Choice Questions) – 1 Mark

1. For some integer m, every even integer is of the form:

   (A) m (B) m + 1 (C) 2m (D) 2m + 1

   Ans. (C) 2m

2. If p^2 is an even integer, then p is an:

   (A) odd integer (B) even integer (C) multiple of 3 (D) none of these

   Ans. (B) even integer

3. If HCF of 336 and 54 is 6, then LCM is:

   (A) 3023 (B) 3024 (C) 3025 (D) none of these

   Ans. (b 3024

4. Prime factors of 4050 is:

   (A) $2 \times 3^3 \times 5$ (B) $2 \times 3^4 \times 5$ (C) $2 \times 3^4 \times 5^2$ (D) $2 \times 3^4 \times 5^3$

   Ans. (C) $2 \times 3^4 \times 5^2$

5. Every composite number can be expressed as a product of:

   (A) co-primes (B) primes (C) twin primes (D) none of these

   Ans. (B) primes

6. If two positive integers p and q can be expressed as $p = a^2 b^3$ and $q = a b$; a, b being prime numbers, then LCM (p, q) is:

   (A) a b (B) $a^2 b^2$ (C) $a^2 b^3$ (D) $a^3 b^3$

   Ans. (C) $a^2 b^3$

7. The decimal expansion of a rational number is always:

(A) non- terminating (B) non-terminating and non-repeating

(C) Terminating or non-terminating repeated (D) none of these

Ans. (C) Terminating or non-terminating repeated

8. The product of a non-zero rational number and an irrational number is:

   (A) An irrational number (B) a rational number (C) one (D) zero

   Ans. (A) an irrational number

9. Which of the following is false?

   A. HCF (p, q) X LCM (p, q) = p x q

   B. HCF(p, q, r) = 1; if p, q, r, are prime numbers

   C. LCM (p, q, r) = p x q x r ; if p, q, r, are prime numbers

   D. HCF(p, q, r) X LCM (p, q, r) = p x q x r

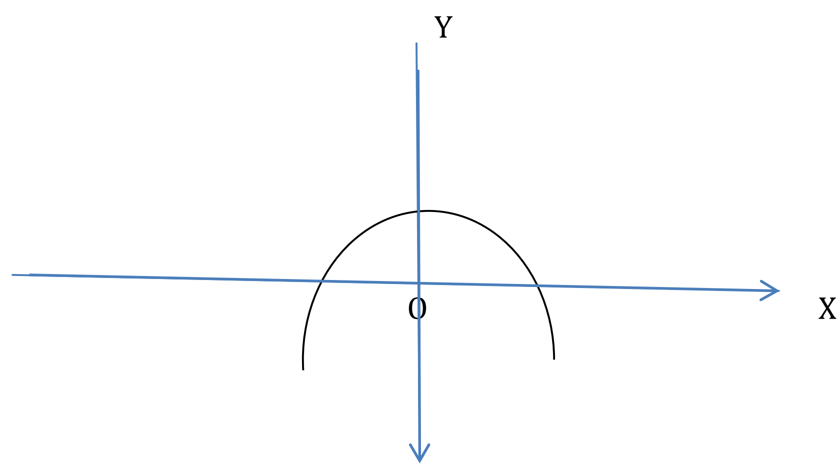
   Ans. (D) HCF(p, q, r) X LCM (p, q, r) = p x q x r

Note: HCF (p, q, r) X LCM (p, q, r) ≠ p x q x r, where p, q, r, are positive integers

10. If HCF of 336 and 54 is 6, then LCM is:

    (a) 2236 (B) 2338 (C) 22338 (D) 757 Ans. (C) 22338

11. The graph of f(x) is shown below. The number of zeroes of f(x) are:



    (A) 1 (B) 2 (C) 3 (D) 4

    Ans.(B) 2

12. The HCF of p and q which are relatively primes is:

    (A)1 (B) p (C) q (D) p q

    Ans. (A) 1

13. The LCM of p and q which are relatively primes is

    (A)1 (B) p (C) q (D) p q

    Ans. (D) p q

14. The product of prime factors of 156 is:

(A) 2 x 3 x 13 (B) 2^2 x 3 x 13 (C) 2 x 3^2 x 13 (D) 2^2 x 3^2 x 13

    Ans. (B) 2^2 x 3 x 13

15. Which of the following is a quadratic polynomial?

(A) $x + 7$ (B) $x^2 - 2$ (C) $x^3 + 4x + 9$ (D) $x^4 + 3x^3 + 2x + 7$

Ans. (B) $x^2 - 2$

16. A polynomial of degree 2 is called a:

(A) Linear polynomial   (B) quadratic polynomial

(C) cubic polynomial   (D) biquadratic polynomial

Ans. (B) quadratics polynomial

17. A quadratic polynomial can have at most:

(A) 1 zero   (B) 2 zeroes   (C) 3 zeroes   (D) 4 zeroes

Ans. (B) 2 zeroes

18. The degree of a constant polynomial is:

(A) 2   (B) 1   (C) -1   (D) 0

Ans. (D) 0

19. The degree of a zero polynomial is:

(A) Always zero   (B) never zero   (C) negative   (D) undefined

Ans. (D) undefined

20. Sum of zeroes of the polynomial $p(x) = x^2 - 3x + 2$ is:

(A) 2   (B) 3   (C) -2   (D) -3

Ans. (B) 3

Hint: sum of zeroes of p(x) = $- \frac{co-effcient\ of\ x}{co-efficient\ of\ x2}$

21. Product of zeroes of the polynomial $p(x) = x^2 - 3$ is:

(A) – 3   (B) 3   (C) $\sqrt{3}$ (D) $-\sqrt{3}$

Ans. (A) – 3

Hint: product of zeroes of p(x) = $\frac{constant\ term}{co-efficient\ of\ x2}$

22. Number of zeroes of a polynomial of degree n is:

(A) Equal to n   (B) greater than n   (C) less than n   (D) less than or equal to n

Ans. (D) less than or equal to n

23. If the graph of y = p(x), where p(x) is a polynomial, does not intersect the x- axis then the number of zero is:

(A) 1   (B) 2   (C) 3   (D) none of the above

Ans. (D) none of the above

24. If the graph of $y = p(x)$ where $p(x)$ is a polynomial, intersects the x- axis at one point only then the number of zero is:

(A)1 (B) 2 (C) 3 (D) 4

Ans. (A) 1

25. If the sum of zeroes of polynomial $p(x) = 2x^2 - k\sqrt{2}\,x + 1$ is $\sqrt{2}$, then the value of k is:

(A) $\sqrt{2}$ (B) $2\sqrt{2}$ (C) 2 (D) ½

Ans.(C) 2

26. At most how many zeroes a linear polynomial can have?

(A)0 (B) 1 (C) 2 (D) 3

Ans.(B) 1

27. If the graph of two lines pass through the same points, then the system of equations representing these lines is:

(A)consistent (B) inconsistent

(C) consistent dependent (D) inconsistent and dependent

Ans. (C) consistent dependent

28. The pair of equations $x = 0$ and $y = 0$ has:

(A)no solution (B) one solution

(C) two solutions (D) infinitely many solutions

Ans. (B) one solution

29. The pair of equations $x = a$ and $y = b$ graphically represents lines which are:

(A) parallel (B) coincident

(C) intersecting at (a, b) (D) intersecting at (b, a)

Ans. (C) intersecting at (a, b)

30. The system of equations $-3x + 4y = 5$ and $\frac{9}{2}x - 6y + \frac{15}{2} = 0$ has:

(A)Unique solution (B) infinite solutions (C) no solutions (D) none of these

Ans. (B) infinite solutions

31. If $x = a$ and $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the vales of a and b are respectively:

(A)3 and 5 (B) 3 and 1 (C) 5 and 3 (D) -1 and -3

Ans. (B) 3 and 1

32. If the roots of the equation $ax^2 + bx + c = 0$ are equal, then c equals to:

(A) $\frac{b}{2a}$ (B) - $\frac{b}{2a}$ (C) $\frac{b^2}{4a}$ (D) - $\frac{b^2}{4a}$

Ans. (C) $\frac{b^2}{4a}$

33. The roots of the equation $ax^2 + b x + c = 0$ are not-real if:

(A) $b^2 - 4ac = 0$ (B) $b^2 - 4ac > 0$ (C) $b^2 - 4ac < 0$ (D) $b = 0$

Ans. (C) $b^2 - 4ac < 0$

34. The sum of a number and its reciprocal is $\frac{10}{3}$ then the number is:

(A) 1 (B) 2 (C) 3 (D) 4

Ans. (C) 3

35. If x = 1 is the common root of $ax^2 + b x + 2 = 0$ and $x^2 + x + b = 0$, then ab equals:

(A) 1 (B) 0 (C) 3 (D) 4

Ans. (B) 0

36. A quadratic equation $ax^2 + b x + c = 0$, $a \neq 0$ will have equaled roots if:

(A) $b^2 - 4ac = 0$ (B) $b^2 - 4ac > 0$ (C) $b^2 - 4ac < 0$ (D) $b = 0$

Ans. (A) $b^2 - 4ac = 0$

37. The roots of the quadratic equation $x^2 + 4x + 4 = 0$ is:

(A) (2, 0) (B) (2, 2) (C) (-2, - 2) (D) (2, -2)

Ans. (C) (-2, -2)

38. The system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2 y + c_2 = 0$ has an infinite number of Solutions if:

(A) $a_1/a_2 \neq b_1/b_2$ (B) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ (C) $a_1/a_2 = b_1/b_2 = c_1/c_2$ (D) none of these

Ans. (C) $a_1/a_2 = b_1/b_2 = c_1/c_2$

39. The system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2 y + c_2 = 0$ has a unique solution if:

(A) $a_1/a_2 \neq b_1/b_2$ (B) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ (C) $a_1/a_2 = b_1/b_2 = c_1/c_2$ (D) none of these

Ans. (A) $a_1/a_2 \neq b_1/b_2$

40. The system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2 y + c_2 = 0$ has no solution if:

(A) $a_1/a_2 \neq b_1/b_2$ (B) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ (C) $a_1/a_2 = b_1/b_2 = c_1/c_2$ (D) none of these

Ans. (B) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

41. In an AP, if d= 2, n = 4, $a_n = 13$ then a, is:

(A) 6 (B) 7 (C) 20 (D) 28

Ans. (B) 7

42. In an AP, if a= 3, d = 2, n = 10 then a_n, is:

(A) 21 (B) 23 (C) 54 (D) 100

Ans. (A) 21

43. Which of the following series form an AP?

(A) 2, 4, 8, 16, ……… (B) 0, -4, -8, -12,……… (C) 1, 3, 9, 27, …….. (D) 0.2, 0.22, 0.222, 0.2222,…..

Ans. (B) 0, -4, -8, -12,……… *forms an AP (as d = -4 – 0 = -4; - 8 –(-4) = -4; -12 –(-8) = -4)*

44. The first three terms of an AP when the first term, a= 5 and common difference, d = 5 are:

(A) 5, 10, 15 (B) 10, 20, 30 (C) 10, - 10, - 20 (D) 10, 5, 0

Ans. (A) 5, 10, 15

45. If common difference of an AP is 5, then $a_{16} - a_{15}$ is:

(A) 1 (B) 31 (C) 5 (D) 15

Ans. (C) 5

46. Sum of first 20 natural numbers is:

(A) 210 (B) 120 (C) 55 (D) 15

Ans. (A) 210

47. Which term in the AP: 68, 64, 60, ……… is -8?

(A) 50^{th} (B) 45^{th} (C) 30^{th} (D) 20^{th}

Ans. (C) 20^{th}

48. The missing terms in the box of the AP: □, 13, □, 3,……. is.

(A) 18, 8 (B) 14, 16 (C) 16, 10 (D) 18, 10

Ans. (A) 18, 8

49. If k, 2k – 1, k + 1 are three consecutive terms of an AP, then the value of k is:

(A) – 2 (B) 3/2 (C) – 3/2 (D) 6

Ans. (B) 3/2

50. If the 5n + 3 is the nth term of an AP, then the common difference is:

(A) 15 (B) 12 (C) 5 (D) 1

Ans. (C) 5

51. The first three terms of an AP if $a_n = 2n + 5$ are:

(A) 1, 2, 3 (B) 7, 10, 13 (C) 7, 9, 11 (D) 11, 13, 17

Ans. (C) 7, 9, 11

52. The sum of first n natural numbers is:

(A) n(n + 1)/2 (B) n(n + 1) (C) $n^2 + 2n/2$ (D) n^3

Ans. (A) $n(n + 1)/2$

53. Two geometrical figures are said to be similar if:

(A) They have same shape and size (B) they have same shape but different sizes

(C) They have same size but different shapes (C) they have same shape

Ans. (A) They have same shape and size

54. All geometrical congruent figures are:

(A) Not similar (B) similar (C) unequal (D) none of the above

Ans. (B) similar

55. The ratio of any two corresponding sides in two equiangular triangles is always:

(A) The same (B) different (C) equal (D) none of the above

Ans. (A) the same

56. If two angles of one triangle are respectively equal to two angles of another triangles then the two triangles are similar. This is referred to as the:

(A) AA Similarity Criterion for two triangles (B) SAS Similarity Criterion for two triangles

(C) AAA Similarity Criterion for two triangles (D) SSS Similarity Criterion for two triangles

Ans. (A) AA Similarity Criterion for two triangles

57. The perimeter of a circle is called the:

(A) Area (B) diameter (C) radius (D) circumference

Ans. (D) circumference

58. The distance of any point P (x, y) from Origin is:

(A) $\sqrt{- x^2 – y^2}$ (B) $\sqrt{- x^2 + y^2}$ (C) $\sqrt{x^2 + y^2}$ (D) $\sqrt{x^2 - y^2}$

Ans. (C) $\sqrt{x^2 + y^2}$

59. In which quadrant does the point (-3 , 5) lie?

(A) first quadrant (B) second quadrant (C) third quadrant (D) fourth quadrant

Ans. (B) second quadrant

60. The distance of the point P (3, 4) from the origin is:

(A) 1 unit (B) 3 units (C) 5 units (D) 7 units

Ans. (C) 5 units

61. The coordinates of the midpoint of the line segment joining the points A(- 2, 8) and B(- 6, -4) is:

(A) (4, 2) (B) (- 4, 2) (C) (4, - 2) (D) (- 4, - 2)

Ans. (B) (- 4, 2)

62. The coordinates of the midpoint of the line segment joining the points A(a, b) and B(0, 0) is:

(A) (a + b/2, a) (B) (a + b, b) (C) (a/2, b/2) (D) (a, b)

Ans. (C) (a/2, b/2)

63. The value of $1 + \tan^2 45^0$ is:

(A) - 1 (B) 0 (C) 1 (D) 2

Ans. (D) 2

64. If cosθ =1, then the value of θ is:

(A) 0^0 (B) 30^0 (C) 60^0 (D) 90^0

Ans. (D) 90^0

65. The value of $3 \cot^2 A - 3 \text{Cosec}^2 A$ is equal to:

(A) - 3 (B) 0 (C) 3 (D) 1

Ans. (A) – 3

66. In ΔABC right angled at B, if AC = 13cm, BC = 5 cm and AB = 12 cm then sin A is equal to:

(A) 13/5 (B) 5/13 (C) 12/13 (D) 13/12

Ans. (B) 5/13

67. The value of $9 \sec^2\theta - 9 \tan^2\theta$ is:

(A) 0 (B) 1 (C) 9 (D) 10

Ans. (C) 9

68. The value of $\text{cosec}^2 (90^0 - \theta) – \tan^2\theta$ is:

(A) 0 (B) 1 (C) 2 (D) 3

Ans. (B) 1

69. Sin 2A = 2 sin A is true when A equals to:

(A) 0^0 (B) 30^0 (C) 45^0 (D) 60^0

Ans. (A) 0^0

70. If tangents PA and PB from a point P to a circle with Centre O are inclined to each other at an angle of 80^0 then <POA is equal to:

(A) 50^0 (B) 60^0 (C) 70^0 (D) 80^0

ANS. (A) 50^0

71. From a point Q, the length of the tangent to a circle is 4 cm and the distance of Q from the Centre is 5 cm, then the radius of a circle is:

(A) 1 cm (B) 5 cm (C) 3 cm (D) 4 cm

Ans. (C) 3 cm

72. The tangent drawn at the end points of a diameter of a circle C are:

(A) Equal (B) parallel (C) perpendicular (D) intersecting

Ans. (B) parallel

73. The distance between two parallel tangents of a circle of radius 8 cm is:

(A) 8 cm (B) 12 cm (C) 14 cm (D) 16 cm

Ans. (D) 16 cm

74. The portion (or part) of a circular region enclosed between a chord and the corresponding arc is called a/an:

(A) Arc of the circle (B) perimeter of a circle (C) sector of a circle (D) segment of a circle.

Ans. (D) segment of a circle

75. The portion (or part) of the circular region enclosed by two radii and the corresponding arc is called the/an:

(A) Arc of the circle (B) perimeter of a circle

(C) Sector of a circle (D) segment of a circle

Ans. (C) sector of a circle

76. The Area of a circle is 49π cm^2. Its circumference is:

(A) 7π cm (B) 14π cm (C) 21π cm (D) 28π cm

Ans. (B) 14π cm

77. In a circle of radius 21cm, an arc subtends an angle of 60^0 at the centre. The length of an arc is: (Take $\pi = 22/7$)

(A) 22 cm (B) 44 cm (C) 132 cm (D) 231 cm

Ans. (A) 21 cm

78. If the area and circumference of a circle are numerically equal, then its diameter is:

(A) 2 units (B) 3 units (C) 4 units (D) 6units

Ans. (C) 4 units

79. If the circumference of a circle increases from 2π to 4π, then its area is:

(A) four times (B) tripled (C) doubled (D) halved

Ans. (A) four times

80. The total surface area of a hemispherical object of radius r, is:

(A) πr^2 sq. units (B) $2\pi r^2$ sq. units (C) $3\pi r^2$ sq. units (D) $4\pi r^2$ sq. units

Ans. (C) $3\pi r^2$ sq. units

81. The length of a diagonal of a cube of side ‘a’ is:

(A) a √3 units (B) 3 √a units (C) √3a units (D) a/√3 units

Ans. (C) a √3 units

82. The area of square is the same as the area of circle. Their perimeters are in the ratio:

(A) 1:1 (B) 2:π (C) π: 2 (D) 2: √π

Ans. (D) 2:√π

83. A Bicycle wheel makes 1000 revolutions in moving 88000 m. the diameter of a wheel is:

(A) 14 m (B) 24 m (C) 28 m (D) 40 m

Ans. (C) 28 m

84. A garden roller has circumference of 4 m. the number of revolutions it makes in moving 40 m is:

(A) 8 (B) 10 (C) 12 (D) 16

Ans. (B) 10

85. Area of a sector angle p^0 of a circle with radius R units is:

(A) p/180 X 2 π R (B) p/180 X π R^2 (C) p/360 X 2π R (D) p/360 X π R^2

Ans. (D) p/360 X π R^2

86. The volume of a sphere of radius R is:

(A) 2/3 π R^3 (B) 4/3π R^3 (C) 3π R^3 (D) 4π R^3

Ans. (B) 4/3π R^3

87. The surface area of a sphere is 616cm^2, its diameter is:

(A) 56 cm (B) 28 cm (C) 14 cm (D) 7 cm

Ans. (C) 14 cm

88. During the conversion of a solid from one shape to another, the volume of the new shape will:

(A) Increase (B) decrease (C) remain unaltered (D) be doubled

Ans. (C) remain unaltered

89. If the radius of a sphere becomes 3 times, then its volume will becomes:

(A) 3 times (B) 6 times (C) 9 times (D) 27 times

Ans. (D) 27 times

90. Thee surface areas of two spheres are in the ratio 16:9. The ratio of their volume is:

(A) 64:27 (B) 16:9 (C) 4:3 (D) $16^3:9^3$

Ans. (A) 64:27

91. A metallic sphere of radius 9 cm is melted to form a solid cylinder of radius 9 cm. the height of the cylinder is:

(A) 12 cm (B) 18 cm (C) 36 cm (D) 96 cm

Ans. (A) 12 cm

92. If volumes of two spheres are in the ratio 64:27, then the ratio of their surface areas is:

(A) 3:4 (B) 16:9 (C) 9:16 (D) 4:3

Ans. (B) 16:9

93. The empirical relationship between mean, mode and median in asymmetrical distribution is:

(A) Mode = 3 Median – 2 Mean (B) Mode = 3 Median + 2 Mean

(C) Mode = 2 Median – 3 Mean (D) Mode = Median – 2 Mean

Ans. (A) Mode = 3 Median – 2 Mean

94. Mode is:

(A) Least frequent value (B) middle most value

(C) Most frequent value (D) none of the above

Ans. (C) most frequent value

95. The cumulative frequency table is useful in determining the:

(A) Mean (B) Median (C) Mode (D) All of the above

Ans. (B) Median

96. If the mode of 16, 15, 17, 16, 15, x, 19, 17, 14 is 15 then x is:

(A) 15 (B) 16 (C) 17 (D) 19

Ans. (A) 15

97. Which of the following cannot be the probability of an event?

(A) 2/3 (B) – 1.5 (C) 0.15 (D) 0.7

Ans. (B) – 1.5

98. A die is thrown once. The probability of getting a number less than 3 is:

(A) 1/2 (B) 1/3 (C)1/4 (D) 1/6

Ans. (B) 1/3

99. A die is thrown once. The probability of getting a prime number is:

(A) 1/2 (B) 1/3 (C) 2/3 (D) 1/6

Ans. (A) 1/2

100. The probability of a sure Event is:

(A) 0 (B) 1/2 (C) 1 (D) non-existent

Ans. (C) 1

101. The probability of an impossible Event is:

(A) 0 (B) 1/2 (C) 1 (D) non-existent

Ans. (A) 0

102. If P(E) = 0.05, then P(E') equals:

(A) - 0.05 (B) 0.5 (C) 0.9 (D) 0.95

Ans. (D) 0.95

103. A jar contains 6 red, 5 black, and 3 green marbles of equal size. The probability that a randomly drawn marble would be green in colour is:

(A) 5/14 (B) 11/14 (C) 3/14 (D) 6/14

Ans. (C) 3/14

104. The sum of the values of all the observations divided by the total number of observations is called:

(A) Mean (B) mode (C) median (D) frequency

Ans. (A) mean

105. In a frequency distribution, the class having the maximum frequency is called:

(A) Class mark (B) class size (C) median class (D) modal class

Ans. (D) modal class

106. Two coins are tossed. The probability of getting atmost one head is:

(A) 1 (B) 3/4 (C) 1/2 (D) ¼

Ans. (B) 3/4

107. The perimeter of a circle of radius 7 units is: (take π = 22/7)

(A) 7 units (B) 22 units (C) π units (D) 44 units

Ans. (D) 44 units

108. The angle described by a minute hand in 1 hour is:

(A) 60^0 (B) 120^0 (C) 180^0 (D) 360^0

Ans. (D) 360^0

109. The circumference of a circle of diameter d units is:

(A) π d (B) 2πd (C) 4π/2 (D) π d/2

Ans. (A) π d

110. If the probability of a player winning a game is 0. 79, then the probability of his losing the same game is:

(A) 1. 79 (B) 0. 31 (C) 0. 21% (D) 0. 21

Ans. (D) 0. 21

111. If the roots of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ are real and equal, then which of the following relation is true?

(A) $a = b^2/c$ (B) $b^2 = ac$ (C) $ac = b^2/4$ (D) $c = b^2/a$

Ans. (C) $ac = b^2/4$

112. If $\cos(\alpha + \beta) = 0$, then the value of $\cos((\alpha + \beta)/2)$ is equal to:

(A) $1/\sqrt{2}$ (B) $1/2$ (C) 0 (D) $\sqrt{2}$

Ans. (A) $1/\sqrt{2}$

113. For some data $x_1, x_2, \ldots\ldots x_n$ wth respective frequencies $f_1, f_2, \ldots\ldots f_n$, the value of $\sum f_i (x_i - x')$is equal to:

(A) nx' (B) 1 (C) $\sum f_i$ (D) 0

Ans. (D) 0

114. The middlemost observation of every data arranged in order is called:

(A) Mode (B) median (C) mean (D) deviation

Ans. (B) median

115. Every composite number can be expressed as product of primes and this factorisation is unique, apart from the order in which the prime factors occur. This is known as:

(A) Pythagoras theorem (b Euclid's Division algorithm

(C) Fundamental Theorem of Arithmetic (D) Thales's theorem

Ans. (C) Fundamental Theorem of Arithmetic

116. A number which cannot be expressed in the form a/b, where 'a' and 'b' are both integers and $b \neq 0$ is called a/an:

(A) Rational number (B) irrational number (C) composite number (D) prime number

Ans. (B) An irrational number

117. A number which can be expressed in the form a/b, where 'a' and 'b' are both integers and $b \neq 0$ is called a/an:

(A) Rational number (B) irrational number (C) composite number (D) prime number

Ans. (A) A Rational number

118. A number which is not divisible by 2 is called a/an:

(A) Even natural number (B) whole number (C) odd natural number (D) prime number

Ans. (C) An odd natural number

119. A natural number which has exactly two factors i. e., 1 and the number itself is called a:

(A) Rational l number (B) whole number (C) composite number (D) prime number

Ans. (D) prime number

120. A natural number which is not prime and has more than two factors is called a/an:

(A) composite number (B) whole number (C) odd natural number (D) prime number

Ans. (A) a composite number

121. The zero of a linear polynomial P (x) = a x + b, where a, b are real numbers, is:

(A) - a/b (B) – b/a (C) – (ab) (D) a/b

Ans. (B) – b/a

122. For any polynomial p (x), if p (a) =0, then 'a' is called:

(A) Constant of the polynomial (B) zero of the polynomial

(C) Degree of the polynomial (D) coefficient of the polynomial

Ans.(B) zero of the polynomial

123. If 5 pencils and 7 pen together cost ₹ 50, whereas 7 pencils and 5 pens together costs ₹ 46, then the cost of one pen is:

(A) ₹ 3 (B) ₹ 7 (C) ₹ 5 (D) ₹ 12

Ans. (C) ₹ 5

124. If 10 students of class X took part in a Mathematics challenge during the Talent Fest organized by the School, and if the number of girls is 4 more than the number of boys, then the number of boys is:

(A) 3 (B) 4 (C) 5 (D) 6

Ans. (A) 3

125. The Discriminant of the quadratic equation $x^2 + 8x + 16 = 0$ is:

(A) 3 (B) 2 (C) 1 (D) 0

Ans. (D) 0

126. The solution of (x – 4) (x + 2) = 0 is:

(A) 4, - 2 (B) – 4, 2 (C) 2, - 2 (D) 4, 2

Ans. (A) 4, - 2

127. The value of sec 60^0 is:

(A) $\sqrt{3}/2$ (B) 1/2 (C) 2 (D) 1

Ans. (C) 2

128. The area of an equilateral triangle of side 'a' is:

(A) $3a^2$ (B) $2\sqrt{3}/a$ (C) $\sqrt{3}/4$ a (D) $\sqrt{3}/4\ a^2$

Ans. (D) $\sqrt{3}/4\ a^2$

129. The length of the altitude of an equilateral triangle of side 2 cm is:

(A) 3 (B) $\sqrt{3}$ (C) $\sqrt{3}/2$ (D) $2\sqrt{3}$

Ans. (B) $\sqrt{3}$

130. If $\Delta ABC \sim \Delta DEF$ and $< A = 47^0$ and $<E = 83^0$, then $<C$ is:

(A) 50^0 (B) 47^0 (C) 80^0 (D) 83^0

Ans. (A) 50^0

131. In a triangle, if the perpendicular from the vertex to the base bisects the base. The triangle is:

(A) Scalene (B) isosceles (C) obtuse –angled (D) right- angled

Ans. (B) isosceles

132. If the bisector of an angle of a triangle bisects the opposite side, then the triangle is:

(A) Scalene (B) isosceles (C) equilateral (D) right- angled

Ans. (C) Equilateral

133. If A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the three vertices of a triangle then area of a ΔABC is:

(A) $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$ (B) $\frac{1}{2}|x_2(y_3 - y_1) + x_3(y_1 - y_2) + x_3(y_1 - y_2)|$

(C) $\frac{1}{2}|x_3(y_2 - y_1) + x_2(y_3 - y_1) + x_1(y_3 - y_2)|$ (D) $\frac{1}{2}|x_1(y_2 - y_1) + x_2(y_3 - y_2) + x_3(y_1 - y_3)|$

Ans. (A) $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

134. Three points in a plane are collinear if the area of a triangle is:

(A) 1 (B) 1/2 (C) -1 (D) 0

Ans. (D) 0

135. Which one of the following is the Hero's Formula for finding the area of a triangle?

(A) ½ base X height (B) $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

(C) $\sqrt{s(s-a)(s-b)(s-c)}$, where a , b, c are the sides of a triangle and s = a + b + c/2

(D) None of the above

Ans. (C) $\sqrt{s(s-a)(s-b)(s-c)}$, where a , b, c are the sides of a triangle and s = a + b + c/2

136. The relation between $\sin\theta$, $\cos\theta$ and $\tan\theta$ is:

(A) $\cos\theta/\sin\theta = \tan\theta$ (B) $\sin\theta/\cos\theta = \tan\theta$

(C) $\tan\theta/\sin\theta = \cos\theta$ (D) $\tan\theta/\cos\theta = \sin\theta$

Ans. (B) $\sin\theta/\cos\theta = \tan\theta$

137. The relation between sinθ, cosθ and cotθ is:

(A) $\cos\theta/\sin\theta = \tan\theta$ (B) $\sin\theta/\cos\theta = \tan\theta$

(C) $\tan\theta/\sin\theta = \cos\theta$ (D) $\tan\theta/\cos\theta = \sin\theta$

Ans. (A) $\cos\theta/\sin\theta = \cot\theta$

138. If $\theta = 30^0$ then the value of $\cos^2\theta - \sin^2\theta$ is:

(A) 1 (B) 1/2 (C) -1 (D) 0

Ans. (B) 1/2

139. A part of the circle whose end points are end point of a diameter is called a:

(A) Circumference (B) segment (C) semicircle (D) perimeter

Ans. (C) semicircle

140. A wire is bent in the form of a circle of radius 28 cm, and then its length is:

(A) 176 cm (B) 188 cm (C) 228 cm (D) 236 cm

Ans. (A) 176 cm

141. In a circle of radius 21 cm, an arc subtends an angle of 60^0 at the centre. Then the length of the arc is:

(A) 20cm (B) 21 cm (C) 22 cm (D) 121 cm

Ans. (C) 22 cm

142. If 'r' and 'h' represent the radius of the base and height of a right circular cone respectively then its curved surface area is:

(A) π r h units (B) $\pi r^2 h$ units (C) $\pi r (\sqrt{} r^2 + h^2)$ units (D) $\pi r h^2$ units

Ans. (C) $\pi r (\sqrt{} r^2 + h^2)$ units

143. The mean of first 10 natural numbers is:

(A) 5.5 (B) 9.5 (C) 10.5 (D) 11.5

Ans. (A) 5.5

144. The mean of first 10 whole numbers is:

(A) 5.5 (B) 4.5 (C) 3.5 (D) 7.5

Ans. (A) 5.5

145. The point (3, - 4) lies in the:

(A) First quadrant (B) second quadrant (C) third quadrant (D) fourth quadrant

Ans. (D) fourth quadrant

146. The volume of a cube of an edge of 4 cm is:

(A) 16 cm^3 (B) 64 cm^3 (C) 128 cm^3 (D) 256 cm^3

Ans.(B) 64 cm^3

147. Eight metallic sphere each of radius 2mm are melted and recast into a single sphere. Then the radius of the new sphere is:

(A) 4 mm (B) 4.5 mm (C) 5.5 mm (D) 6 mm

Ans. (A) 4 mm

148. The difference between the minimum and maximum values of the data is called:

(A) class limits (B) class interval (C) class size (D) range of data

Ans.(D) range of data

149. A die is thrown once. The probability of getting an even number is:

(A) 1/4 (B) 2/3 (C) 1/2 (D) 1/3

Ans. (C) ½

150. A letter in English alphabets is chosen at random then the probability that the chosen letter is a consonant is:

(A) 21/26 (B) 11/25 (C) 15/26 (D) 21/23

Ans. (A) 21/26

151. A bi-quadratic polynomial is a polynomial of degree:

(A) 1 (B) 2 (C) 3 (D) 4

Ans. (D) 4

152. If $b^2 - 4ac > 0$, then the quadratic equation $ax^2 + bx + c = 0$ has:

(A) real and equal roots (B) real and unequal roots

(C) no real roots (D) none of the above

Ans. (B) real and unequal roots

153. If $b^2 - 4ac = 0$, then the roots of the quadratic equation $ax^2 + bx + c = 0$ are:

(A) - b/2a, - b /2a (B) –b/2a, b/2a (C) 2b/a, – 2b/a (D) a/2b, - a/2b

Ans. (A) - b/2a, -b /2a

154. In an A.P: a, a+ d, a + 2d, a + 3d, ………., its general term t_n equals to:

(A) a + n d (B) {a + (n - 1)d} (C) (a + n)/2 (D) {n (a + n)d}

Ans. (B) {a + (n - 1)d}

155. The total surface area of a cube of length ‘a’ units is:

(A) $3a^2$ sq. units (B) $4a^2$ sq. units (C) $6a^2$ sq. units (D) $8a^2$ sq. units

Ans. (C) $6a^2$ sq. units

156. The total surface area of a right circular cylinder of radius 'r' units ad height 'h' units is:

(A) sq. units (B) 2π r h sq. units (C) 2π r(r - h) sq. units (D) 3π r (r +h) sq. units

Ans. (A) 2π r (r +h) sq. units

157. The coordinates of the point P (x, y) which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$in the ratio m:n is (x, y) equals to:

(A) $\left(\frac{mx_2+mx_1}{m+n}, \frac{my_2+my_1}{m+n}\right)$ (A) $\left(\frac{mx_2-mx_1}{m+n}, \frac{my_2-my_1}{m+n}\right)$

(A) $\left(\frac{mx_2-mx_1}{m-n}, \frac{my_2-my_1}{m-n}\right)$ (A) $\left(\frac{mx_2+mx_1}{m-n}, \frac{my_2+my_1}{m-n}\right)$

Ans. (A) $\left(\frac{mx_2+mx_1}{m+n}, \frac{my_2+my_1}{m+n}\right)$

158. If the mean of a frequency distribution is 8.1, and $\sum f_i x_i = 132 + 5h$, $\sum f_i = 20$, then h equals:

(A) 3 (B) 4 (C) 5 (D) 6

Ans. (D) 6

159. A die is thrown once, then the probability of getting an even prime number is:

(A) 1/6 (B) 1/2 (C) 1/3 (D) 2/3

Ans. (A) 1/6

160. If sin A = 3/5, then the value of tan A is:

(A) 4/3 (B) 3/4 (C) 3/5 (D) 5/4

Ans. (B) ¾

161. if α, β and γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, where a, b, c, d are real numbers ad a ≠ 0, then the value of αβ + βγ + γα is equal to:

(A) - d/a (B) – c/a (C) b/a (D) c/a

Ans. (D) c/a

162. If 'a' is the first term, 'd' the common difference of an A P, then sum of first n terms is:

(A) n/2 {(a + (n - 1) d} (B) n/2 {(2a + n) d}

(C) n/2 {2a + (n -1) d} (D) n { 2a + (n - 1) d}

Ans. (C) n/2{ 2a + (n - 1) d}

163. If 'a' is the first term, l is the last term of an A P, then its nth sum is:

(A) n/2 {(a + l} (B) n/2 {(2a + l}

(C) n/2 {2a - l} (D) n { a + l}

Ans. (A) n/2{ a + l}

164. In an AP, the difference between t_{n+1} and t_n is called the:

(A) First term (B) last term (C) common difference (D) next term

Ans. (C) common difference

165. In a triangle, the line segment joining from one vertex to the mid -point of the opposite side is called its:

(A) Median (B) perpendicular (C) hypotenuse (D) angle bisector

Ans. (A) median

166. Centroid of a triangle is the point of concurrency of its three:

(A) Angle bisectors (B) medians (C) altitudes (D) perpendicular bisectors

Ans. (B) medians

167. When the co-ordinate axes intersect each other at a point called Origin, its coordinates are:

(A) (x, y) (B) (- x, - y) (C) (0. 0) (D) (- x, y

Ans. (C) (0, 0)

168. The reciprocal of cosine θ is:

(A) tan θ (B) sec θ (C) cosec θ (D) sin θ

Ans. (D) sin θ

169. $\tan^2\theta + 1$ is equal to:

(A) $\cot^2\theta$ (B) $\sec^2\theta$ (C) $\text{cosec}^2\theta$ (D) $\cos^2\theta$

Ans. (B) $\sec^2\theta$

170. $1 - \sin^2\theta$ is equal to:

(A) $\cot^2\theta$ (B) $\sec^2\theta$ (C) $\text{cosec}^2\theta$ (D) $\cos^2\theta$

Ans. (D) $\cos^2\theta$

171. The value of $\text{cosec}^2 45^0$ is:

(A) $1/\sqrt{2}$ (B) $1/\sqrt{3}$ (C) 2 (D) $\sqrt{2}$

Ans. (C) 2

172. The coordinates of centroid of a triangle with vertices (x_1, y_1), (x_2, y_2) and $(x_3, y_3)$are:

(A) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ (A) $\left(\frac{x_1-x_2+x_3}{3}, \frac{y_1-y_2+y_3}{3}\right)$

(A) $\left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}\right)$ (A) $\left(\frac{x_1+x_2-x_3}{3}, \frac{y_1+y_2-y_3}{3}\right)$

Ans. (A) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

173. The coordinates of a mid- point of the line segment AB with end points A (x_1, y_1), B (x_2, y_2) is:

(A) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ (A) $\left(\frac{x_1-x_2}{2}, \frac{y_1+y_2}{2}\right)$

(A) $\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right)$ (A) $\left(\frac{x_1+x_2}{2}, \frac{y_1-y_2}{2}\right)$

Ans. (A) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

174. The perpendicular distance of a point from Y- axis called:

(A) Ordinate (B) abscissa (C) altitude (D) none of the above

Ans. (B) abscissa

175. The ordinates of all points on a horizontal line are:

(A) Parallel (B) perpendicular (C) equal (D) coincident

Ans. (C) equal

176. The abscissa of any point on the Y – axis is:

(A) 1 (B) – 1 (C) 0 (D) 2

Ans. (C) 0

177. The distance between the points P (a, 0) and Q (0, b) is:

(A) $\sqrt{a^2 + b^2}$ (B) a + b (C) $\sqrt{a^2 - b^2}$ (D) a – b

Ans. (A) $\sqrt{a^2 + b^2}$

178. The distance between the points (- 8/5, 2) and (2/5, 2) is:

(A) 1unit (B) 2 units (C) 3 units (D) 4 units

Ans. (B) 2 units

179. The distance between the points A (4, k) and B (1, 0) is 5 units then k equals:

(A) 4 (B) – 4 (C) 0 (D) ± 4

Ans. (D) ± 4

180. The distance between the points P (0, 5) and Q (- 5, 0) is:

(A) 5 units (B) $5\sqrt{2}$ units (C) $2\sqrt{5}$ units (D) $\sqrt{10}$ units

Ans. (B) $5\sqrt{2}$ units

181. If the end points of a diameter of a circle are (1, 2) and (3, 4) then the coordinates of the Centre are:

(A) (2, 4) (B) (2, 3) (C) (1, 2) (D) (4, 6)

Ans. (B) (2, 3)

182. The coordinates of reflection of the point P (- 1, - 3) in X –axis are:

(A) (- 1, 3) (B) (1, 3) (C) (1, - 3) (D) none of the above

Ans. (A) (- 1, 3)

183. Distance covered by a wheel in one revolution is equal to:

(A) Diameter of a wheel (B) area of a wheel

(C) Circumference of a wheel (D) none of these

Ans. (C) circumference of a wheel

184. The circumference of a bicycle wheel that makes 5000 revolutions in moving 11 km is:

(A) 500 cm (B)250 cm (C) 220 cm (D) 150 cm

Ans. (C) 220 cm

185. Angle described by the hour hand of a clock in 12 hours is:

(A) 360^0 (B) 180^0 (C) 270^0 (D) 90^0

Ans. (A) 360^0

186. The perimeter of a scalene triangle having sides 15 cm, 14 cm, 13 cm is:

(A) 42 cm (B)52 cm (C) 72 cm (D) 84 cm

Ans. (A) 42 cm

187. The perimeter of an equilateral triangle with side 9 cm is:

(A) 9 cm (B)18 cm (C) 27 cm (D) 36 cm

Ans. (C) 27 cm

188. If the radius of a sphere is doubled then the ratio of the volumes of the first sphere to the new sphere is:

(A) 1:2 (B)1:4 (C) 1:6 (D) 1:8

Ans. (D) 1:8

189. The sum and product of the zeroes of a quadratic polynomial $x^2 - 9x + 14$ are:

(A) 9, 14 (B) – 9, 14 (C) 9, - 14 (D) 14, 9

Ans. (A) 9, 14

190. The sum and product of the zeroes of a quadratic polynomial $k^2x^2 - k x + 1$ are:

(A) $1/k, - 1/k$ (B) $- 1/k, 1/k^2$ (C) $1/k, 1/k^2$ (D) $1/k, - 1/k^2$

Ans. (C) $1/k, 1/k^2$

191. The zeroes of a quadratic polynomial $x^2 + 9 x - 10$ are:

(A) 4, 5 (B) – 1, 10 (C) 9, -1 (D) -10, 1

Ans. (D) – 10, 1

192. The quadratic polynomial with 2 as sum and – 8 as product of its zeroes is:

(A) $x^2 - 2x - 8$ (B) $x^2 + 2x - 8$ (C) $x^2 - 2x + 8$ (D) $x^2 + 2x + 8$

Ans. (A) $x^2 - 2x - 8$

193. The quadratic polynomial with 0 and – 1/7 as its two zeroes is:

(A) $7x^2 + x$ (B) $x^2 – 7x$ (C) $x^2 + 7x$ (D) $7x^2 – x$

Ans. (A) $7x^2 + x$

194. The first three terms of an AP whose first term 'a' = 1/3 and common difference d = - 2/3 are:

(A) 1/3, - 1/3, - 1 (B) 1/3, 2/3, 1 (C) 1/3, - 1, - 1/3 (D) 1/3, - 1/3, 2/3

Ans. (A) 1/3, - 1/3, - 1

195. The first three terms of an AP whose first term 'a' = 3. 5 and common difference d = 2. 5 are:

(A) 2.5, 6.5, 8.5 (B) 3.5, 6.0, 8.5 (C) 3.5, 5.5, 7.5 (D) 3.5, 6.5, 8.5

Ans. (C) 3.5, 6.0, 8.5

196. If a = - 2, d = 5 then the value of t_{10} is equal to:

(A) 23 (B) 33 (C) 43 (D) 53

Ans. (C) 43

197. The value of t_{15} of an AP: - 3, 5, 13, …… is:

(A) 69 (B) 85 (C) 93 (D) 109

Ans. (D) 109

198. The common difference of the AP: 0, ¼, ½, ¾, ……… is:

(A) 1/4 (B) 1/2 (C) 3/4 (D) 0

Ans. (A) 1/4

199. The quadratic formula of the quadratic equation $ax^2 + bx + c$ where a, b, c are real numbers and a ≠0 is given by x equals to:

(A) $b^2 – 4ac$ (B) $\sqrt{b^2} – 4ac$ (C) $(– b \pm \sqrt{(b^2 – 4ac)})/2$ (D) $-b \pm \sqrt{(b^2 – 4ac)}$

Ans. (C) $(– b \pm \sqrt{(b^2 – 4ac)})/2$

200. If the value of $b^2 – 4ac$, of the quadratic equation $ax^2 + bx + c$ where a, b, c are real numbers and a ≠0 is negative then the nature of its roots are:

(A)not real (B) real and unequal (C) real and equal (D)none of these

Ans. (A) not real

201. If the probability of winning a game is 0.3, then the probability of losing it is:

(A) 1.03 (B) 1.0 (C) 0.9 (D) 0.7

Ans. (D) 0.7

202. A letter is selected at random from the letters of the words 'MATHEMATICS' then the probability of getting the letter M is:

(A) 2/11 (B) 6/11 (C) 4/11 (D) 5/11

Ans. (A) 2/11

203. The mode of the following data:

110,120,130,120,110,140, 130, 120, 140, 120 is:

(A) 140 (B) 130 (C) 120 (D) 110

Ans. (C) 120

204. The median of the following data:

7, 5, 12, 9, 24, 8, 4, 7, 10 is:

(A) 24 (B) 9 (C) 12 (D) 8

Ans. (D) 8

(Hint: arranging the data in ascending order as 4, 5, 7, 7, 8, 9, 10, 12, 14 and apply formula for median when n is odd)

205. The median of the following data:

6, 8, 15, 16, 9, 22, 21, 25, 18 is:

(A) 21 (B) 18 (C) 16 (D) 9

Ans. (C) 16

210. If the mean of 6, 8, 5, 7,4 and x is 7, then x equals to:

(A) 12 (B) 24 (C) 28 (D) 30

Ans. (A) 12

211. If the mean of first n natural numbers is 5n/9, then n is:

(A) 4 (B) 5 (C) 8 (D) 9

Ans. (D) 9

212. If the mean of x, x + 3, x + 6, x + 9 and x + 12 is 10, then x is:

(A) 1 (B) 2 (C) 4 (D) 6

Ans. (C) 4

213. If 10 is the length of the line segment joining the origin from the point P (x, 8), then x is:

(A) 6 (B) 7 (C) 9 (D) 12

Ans. (A) 6

214. A jar contains 25 marble with 10 green marbles and the rest are blue marbles. If a marble is drawn at random from the jar, then the probability that the drawn marble is blue is:

(A) 2/5 (B) 3/5 (C) 4/5 (D) 1/5

Ans. (B) 3/5

215. If A (- 1, 0), B (5, - 2) ad C (8, 2) are the vertices of a triangle ABC, then its centroid is:

(A) (6, 0) (B) (0, 6) (C) (4, 0) (D) (12, 0)

Ans. (C) (4, 0)

216. The probability that a non- leap year has 53 Sundays is:

(A) 1/7 (B) 2/7 (C) 5/7 (D) 6/7

Ans. (A) 1/7

217. The probability that a number selected at random from the numbers 3,4,5,6,7, 8, 9 is a multiple of 4 is:

(A) 1/7 (B) 2/7 (C) 5/7 (D) 6/7

Ans. (B) 2/7

Section-B

Very Short Answer Questions (2 Marks)

Chapter 1: Real numbers

1. Using Euclid's division algorithm, find the HCF of 100 and 190

And Given numbers are 100 and 190

```
∴  100) 190 ( 1
        100
        -----
          90 ) 100 ( 1
                90
               -----
               10 ) 90 ( 9
                    90
                   -----
                     ×
```

By Euclid's division algorithm, we get

$190 = 100 \times 1 + 90$

$100 = 90 \times 1 + 10$

$90 = 10 \times 9 + 0$

∵ Remainder = 0

∴ HCF (100, 190) = 10

2. Express 2025 as a product of its prime factors

Sol.

3	2 0 2 5
3	6 7 5
3	2 2 5
3	7 5
5	2 5
	5

∴ $2025 = 3^4 \times 5^5$

3. HCF of two numbers is 16 and their product is 3072. Find their LCM.

Ans We know,

HCF × LCM = Product of two numbers

$\Rightarrow$ 16 × LCM = 3072

$\Rightarrow$ LCM = 192

4. Find the HCF and LCM of 26 and 91 by prime factorisation method.

Ans

3 2 6   7 9 1

1 3   1 3

$\therefore$ 26 = 2 × 13

91 = 7 × 13

$\therefore$ HCF = 13

$\therefore$ LCM = 13 × 2 × 7 = 182

5. Find the smallest number which when divided by 35, 56 and 91 leaves remainder 7 in each case.

Ans

5 3 5   2 5 6   7 9 1

7   2 | 2 8   1 3

2 | 14

7

$\therefore$ 35 = 5 × 7

56 = 2^3 × 7

91 = 7 × 13

$\therefore$ LCM = 7 × 5 × 2^3 × 13

= 3460

$\therefore$ The smallest number = 3460 + 7

= 3467

6. Find the HCF of 24, 36, 176 by applying the prime factorisation method.

Ans

3 2 4   2 3 6   2 1 7 6

2 | 1 2   2 | 1 8   2 | 8 8

2	6		3	9		2	44
	3			3		2	22
							11

$24 = 2^3 \times 3$

$36 = 2^2 \times 3^2$

$176 = 2^4 \times 11$

$\therefore$ HCF $= 2^4$

$= 4$

7. Find the HCF and LCM of 336 and 54 by applying the fundamental theorem of Arithmetic.

Ans

2	336		2	54
2	168		3	27
2	84		3	9
2	42			3
3	21			
	7			

$\therefore 336 = 2^4 \times 3 \times 7$

$54 = 2 \times 3^3$

$\therefore$ HCF $= 2 \times 3 = 6$

$$LCM = \frac{Product\ of\ two\ numbers}{HCF}$$

$$= \frac{336 \times \cancel{54}^{9}}{\cancel{6}_{1}}$$

$= 3024$

8. Find the largest positive integer which divides 615 and 963 leaving 6 in each case.

Ans The required largest number

= HCF (615 – 6, 963 – 6)

= HCF (609, 957)

$$\begin{array}{r|l} 3 & 609 \\ \hline 7 & 203 \\ \hline & 29 \end{array} \qquad \begin{array}{r|l} 3 & 957 \\ \hline 11 & 319 \\ \hline & 29 \end{array}$$

609 = 3 × 7 × 29

957 = 3 × 11 × 29

∴ HCF = 3 × 29

= 87

∴ The largest positive integer which divides 615 and 963 leaving remainder 6 in each case is 87.

9. Using Euclid's division algorithm, state whether 47 and 149 are co-primes or not

And Given numbers are 47 and 149

∴ 47) 149 (3
141
8) 47 (5
40
7) 8 (1
7
1) 7 (7
7
×

By Euclid's division algorithm, we get

149 = 47 × 3+ 8

47 = 8 × 5 + 7

8 = 7 × 1 + 1

7 = 1 × 7 + 0

∵ Remainder = 0

∴ HCF (47, 149) = 1

Hence, 47 and 149 are co-primes.

10. In a class there are 24 girls and 20 boys. Find the minimum number of books that can be distributed equally among girls and boys.

Ans

2	24	2	20
2	12	2	10
2	6		5
	3		

$\therefore$ $24 = 2^3 \times 3$

$20 = 2^2 \times 5$

$\therefore$ LCM $= 2^3 \times 3 \times 5$

$= 120$

$\therefore$ Minimum number of books that can be distributed equally among girls and boys = 120

Chapter 3: Pairs of Linear equation in Two Variables :-

11. Check whether the given pair of linear equations $2x + 5y = 17, 5x + 3y = 14$ has unique, no solution or infinitely many solutions.

Ans $2x + 5y = 17$

$5x + 3y = 14$

Here,

$a_1 = 2, b_1 = 5, c_1 = -17$

$a_2 = 5, b_2 = 3, c_2 = -14$

$\frac{a_1}{a_2} = \frac{2}{5}$

$\frac{b_1}{b_2} = \frac{5}{3}$

$\frac{c_1}{c_2} = \frac{-17}{-14} = \frac{17}{14}$

$\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ The given pair of linear equations has a unique solution.

12. Determine the value of k for which the given pair of linear equations $2x + 3y - 5 = 0$, $kx - 6y - 8 = 0$ have unique solution.

Ans $2x + 3y - 5 = 0$

$kx - 6y - 8 = 0$

Here,

$a_1 = 2, b_1 = 3, c_1 = -5$

$a_2 = k, b_2 = -6, c_2 = -8$

For unique solution,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{2}{k} \neq \frac{\cancel{3}^{1}}{-\cancel{6}_{2}}$

$\Rightarrow \frac{2}{k} \neq \frac{1}{-2}$

$\Rightarrow k \neq -4$

13. Find the value of k for which the system of linear equations $kx + 2y = 5$ and $3x - y = 10$ has no solution.

Ans $kx + 2y = 5$

$3x - 4y = 10$

Here,

$a_1 = k, b_1 = 2, c_1 = -5$

$a_2 = 3, b_2 = -4, c_2 = -10$

For no solutions,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{3} = \frac{2^1}{-4_2} \neq \frac{\cancel{-5}^{1}}{\cancel{-10}_{2}}$

$\Rightarrow \frac{k}{3} = \frac{1}{-2} \neq \frac{1}{2}$

From the first two, $k = \frac{-3}{2}$

From the last two, $k \neq \frac{3}{2}$

$\therefore \qquad k = \frac{-3}{2}$

14. Find the value of k, for the following system of equations will represent the coincident lines?

$$x + 2y + 7 = 0, \qquad 2x + ky + 14 = 0$$

Ans $\qquad x + 2y + 7 = 0$

$2x + ky + 14 = 0$

Here,

$a_1 = 1, b_1 = 2, c_1 = 7$

$a_2 = 2, b_2 = k, c_2 = 14$

For coincident lines,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \qquad \frac{1}{2} = \frac{2}{2} \neq \frac{\cancel{7}^{1}}{\cancel{14}_{2}}$

From the first two, $\frac{1}{2} = \frac{2}{k}$

$\Rightarrow \qquad k = 4$

15. Solve the following system of linear equations:

$2x + 3y = 0$

$3x + 4y = 5$

Ans $\qquad 2x + 3y = 0 \Rightarrow 2x = -3y \Rightarrow x = -\frac{3y}{2}$ -------------(i)

$3x + 4y = 5$

$\Rightarrow \qquad 3\left(-\frac{3y}{2}\right) + 4y = 5$ [Substituting the value of x from (i)]

$\Rightarrow \qquad \frac{-3y}{2} + 4y = 5$

$\Rightarrow \qquad \frac{-9y+8y}{2} = 5$

$\Rightarrow \quad \frac{-y}{2} = 5$

$\Rightarrow \quad -y = 10$

$\Rightarrow \quad y = -10$

Putting the value of y in eqn (i), we get

$$x = -\frac{3y}{2}$$

$$= \frac{-3 \times (-10)^5}{2_1}$$

$$= 15$$

$\therefore x = 15$

$y = -10$

16. Sum of two numbers is 35 and their difference is 13. Find the numbers.

Ans Let one number be x

$\therefore$ The other number $= 35 - x$

According to the question

$$x - (35 - x) = 13$$

$\Rightarrow x - 35 + x = 13$

$\Rightarrow x + x = 13 + 35$

$\Rightarrow 2x = 48$

$\Rightarrow x = \frac{48^{24}}{2_1}$

$\Rightarrow x = 24$

$\therefore$ One number $= x = 24$

Other number $= 35 - x$

$= 35 - 24$

$= 11$

17. If 2 is added to the numerator of a fraction, it reduces to $\frac{1}{2}$ and if 1 is subtracted from the denominator, it reduces to $\frac{1}{3}$. Find the fraction.

Ans Let the fraction be $\frac{x}{y}$

According to the question,

$\frac{x+2}{y} = \frac{1}{2}$

$\Rightarrow y = 2(x+2)$

$\Rightarrow y = 2x + 4$ -------------(i)

Again,

$\frac{x}{y-1} = \frac{1}{3}$

$\Rightarrow 3x = y - 1$

$\Rightarrow 3x = 2x + 4 - 1$ [using eqn (i)]

$\Rightarrow 3x - 2x = 4 - 1$

$\Rightarrow x = 3$

Putting $x = 3$ in eqn (i), we get

$y = 2x + 4$

$\Rightarrow \mathrm{y} = 2 \times 3 + 4$

$\Rightarrow y = 6 + 4$

$\Rightarrow y = 10$

$\therefore$ The required fraction $= \frac{x}{y} = \frac{3}{10}$

18. 10 students of class 10 took part in Mathematics quiz. If the number of girls is 4 more than the number of boys, represent this situation algebraically.

Ans Let the number of boys by x and the number of girls be y

$\because$ The total number of students is 10

$\therefore x + y = 10$

Also, the number of girls is 4 more than the number of boys

$\therefore y = 4 + x$

19. A father is three times as old as his son. In 12 years time, he will be twice as old as his son. Find the present ages of father and the son.

Ans Let the present age of the son be x years

$\therefore$ The present age of the father be $3x$ years

After 12 years,

Son's age will be $(x + 12)$ yrs

Father's age will be $(3x + 12)$ yrs

According to the question

$$3x + 12 = 2(x + 12)$$

$$\Rightarrow 3x + 12 = 2x + 24$$

$$\Rightarrow 3x - 2x = 24 - 12$$

$$\Rightarrow x = 12$$

$\therefore$ Son's Present age $= x$ yrs $= 12$

Father's present age $= 3x$ yrs $= (3 \times 12)$ yrs $= 36$ yrs

20. Check whether the system of equations $6x + 4y = 2,\ 3x + 2y = 1$ is consistent.

Ans $6x + 4y = 2$

$3x + 2y = 1$

Here,

$$a_1 = 6, b_1 = 4, c_1 = -2$$
$$a_2 = 3, b_2 = 2, c_2 = -1$$
$$\frac{a_1}{a_2} = \frac{6}{3} = 2$$
$$\frac{b_1}{b_2} = \frac{4}{2} = 2$$
$$\frac{c_1}{c_2} = \frac{-2}{-1} = 2$$

$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore$ The given system of linear equations is consistent.

Chapter 4: Quadratic Equations:

21. Check whether $x = \frac{5}{3}$ is a solution of $3x^2 + 10 = 11x$ or not

Ans $\quad$ LHS $= 3x^2 + 10$

$= 3 \times \left(\frac{5}{3}\right)^2 + 10 \quad \left[\text{Putting } x = \frac{5}{3}\right]$

$= 3^1 \times \frac{25}{9_3} + 10$

$= \frac{25}{3} + 10$

$= \frac{25 + 30}{3}$

$= \frac{55}{3}$

RHS $= 11x = 11 \times \frac{5}{3} = \frac{55}{3}$

$\because$ LHS = RHS

$\therefore \; x = \frac{5}{3}$ is a solution of the given equation

22. Determine the value of k if $x = -\frac{5}{3}$ is a solution of the equation $3x^2 + kx + 5 = 0$.

Ans $\quad 3x^2 + kx + 5 = 0$

$\Rightarrow 3 \times \left(\frac{-5}{3}\right)^2 + k \times \left(\frac{-5}{3}\right) + 5 = 0$

$\Rightarrow 3^1 \times \frac{25}{9_3} - \frac{5k}{3} + 5 = 0$

$\Rightarrow \frac{25}{3} - \frac{5k}{3} + 5 = 0$

$\Rightarrow \frac{25 - 5k + 15}{3} = 0$

$\Rightarrow 40 - 5k = 0$

$\Rightarrow 5k = 40$

$\Rightarrow k = 8$

23. The product of two consecutive positive integers is 240. Formulate the quadratic equation whose roots are these integers.

Ans Let the first positive integer be x

$\therefore$ The consecutive positive integer $= (x+1)$

$\therefore$ According to the question,

$$x(x+1) = 240$$

$$\Rightarrow x^2 + x = 240$$

$$\Rightarrow x^2 + x - 240 = 0$$

24. Solve the quadratic equation $x^2 + 3x - 18 = 0$ by factorisation.

Ans $x^2 + 3x - 18 = 0$

$$\Rightarrow x^2 + 6x - 3x - 18 = 0$$

$$\Rightarrow x(x+6) - 3(x+6) = 0$$

$$\Rightarrow (x+6)(x-3) = 0$$

$\therefore$ Either, or,

$x + 6 = 0$ $x - 3 = 0$

$\Rightarrow x = -6$ $\Rightarrow x = 3$

$\therefore x = -6, 3$

25. Write the discriminant of $3x^2 - 2x + 8 = 0$.

Ans The given equation is $3x^2 - 2x + 8 = 0$

Here,

$a = 3, b = -2, c = 8$

$\therefore$ Discriminant $= b^2 - 4ac$

$= (-2)^2 - 4 \times 3 \times 8$

$= 4 - 96$

$= -92$

26. Determine the nature of roots of the quadratic equation $6x^2 + 7x - 10 = 0$

Ans The given equation is $6x^2 + 7x - 10 = 0$

Here,

$a = 6, b = 7, c = -10$

Discriminant $(D) = b^2 - 4ac$

$= (7)^2 - 4 \times 6 \times (-10)$

$= 49 + 240$

$= 289$

$\because D > 0$

$\therefore$ The given equation has real and unequal roots.

27. The product of Amit's (in years) 3 years ago and his age (in years) 7 years later is 56. Formulate a quadratic equation to find his present age.

Ans Let Amit's present age be x yrs

3 years ago, Amit's age was $(x - 3)$ yrs

7 years later, Amit's age will be $(x + 7)$ yrs

According to the question,

$(x - 3)(x + 7) = 56$

$\Rightarrow x(x + 7) - 3(x + 7) = 56$

$\Rightarrow x^2 + 7x - 3x - 21 - 56 = 0$

$\Rightarrow x^2 + 4x - 77 = 0$

28. Does the equation $9x^2 - 12x + 4 = 0$ have both roots equal.

Ans The given equation is $9x^2 - 12x + 4 = 0$

Here,

$a = 9, b = -12, c = 4$

$\therefore$ Discriminant $(D) = b^2 - 4ac$

$= (-12)^2 - 4 \times 9 \times 4$

$= 144 - 144$

$= 0$

$\because D = 0$

$\therefore$ The given equation has both roots equal.

29. Find the value of k for which the equation $3x^2 - 5x + 2k = 0$ has real and equal roots.

Ans The given equation is $3x^2 - 5x + 2k = 0$

Here,

$a = 3, b = -5, c = 2k$

The given equation will have real and equal roots if

Discriminant = 0

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (-5)^2 - 4 \times 3 \times 2k = 0$

$\Rightarrow 25 - 24k = 0$

$\Rightarrow -24k = -25$

$\Rightarrow k = \frac{25}{24}$

30. State the nature of the roots of a quadratic equation if its
   (i) discriminant is greater than zero
   (ii) discriminant is equal to zero

Ans (i) If discriminant is greater than zero, then the roots are real and unequal (distinct)

(ii) If the discriminant is equal to zero, then the roots are real and equal (repeated)

Chapter 6: Triangles

31. When are two triangles said to be similar?

Ans Two triangles are said to be similar if their,

i) Corresponding angles are equal
ii) Corresponding sides in the same ratio (or proportional)

32. $D \text{ and } E$ are points on the sides $AB \text{ and } AC$ respectively of a ΔABC. Determine whether $DE \parallel BC$ or not where $AD = 5.7\ cm, DB = 9.5\ cm, AE = 4.8\ cm \text{ and } EC = 8\ cm$.

Ans $$\frac{AD}{DB} = \frac{5.7\ cm}{9.5\ cm}$$

$$= \frac{3}{5}$$

$$\frac{AE}{EC} = \frac{4.8\ cm}{8\ cm}$$

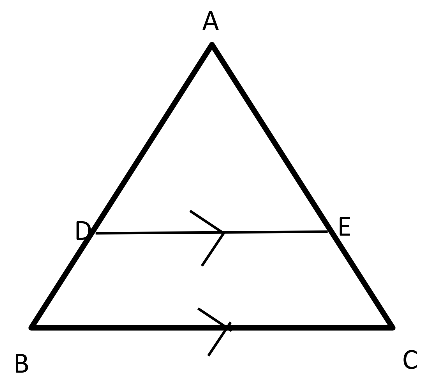
$$= \frac{3}{5}$$

$$\because \frac{AD}{DB} = \frac{AE}{EC}$$

$\therefore$ By the converse of Thales' Theorem (or Basic Proportionality Theore)

$$DE \parallel BC$$

33. In a $\Delta ABC, D \text{ and } E$ are points on the sides AB and AC respectively such that $DE \parallel BC$. If $AD = 2\ cm, AB = 6\ cm, AE = 3\ cm$, find AC.



Ans In ΔABC,

$\because DE \parallel BC$

$\therefore$ By Basic Proportionality Theorem,

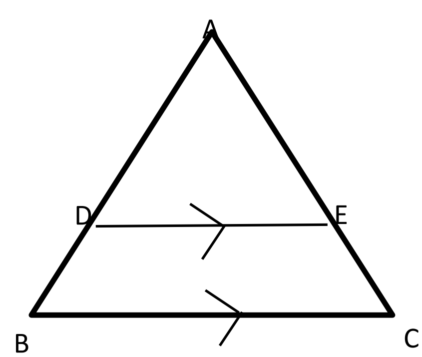
$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\Rightarrow \frac{2}{6} = \frac{3\ cm}{AC}$$

$$\Rightarrow 2 \times AC = 6 \times 3\ cm$$

$$\Rightarrow AC = 9\ cm.$$

34. In a $\Delta ABC, D \text{ and } E$ are points on the sides AB and AC respectively such that $DE \parallel BC$. If $\frac{AD}{DB} = \frac{2}{3}$ and $AE = 7.2cm$, find AC.



Ans In ΔABC

$\because DE \parallel BC$

$\therefore$ By Basic Proportionality Theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{2}{3} = \frac{7.2\ cm}{EC}$$

$\Rightarrow \; EC = 10.8\ cm$

$\therefore AC = AE + EC$

$= 7.2\ cm + 10.8\ cm$

$= 18\ cm$

35. The perimeter of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm, what is the corresponding side of the other triangle.

Ans Let the corresponding side of the other Δ be x

$\because$ the two Δs are similar

$$\therefore \frac{One\ side\ of\ one\ \Delta}{Corresponding\ side\ of\ other\ \Delta} = \frac{Perimeter\ of\ one\ \Delta}{Perimeter\ of\ the\ other\ \Delta}$$

$$\Rightarrow \frac{9\ cm}{x} = \frac{25}{15}$$

$\Rightarrow x \times 25 = 15 \times 9\ cm$

$$\Rightarrow \; x = \frac{27}{5}$$

$\Rightarrow \; x = \; 5.4\ cm$

$\therefore$ The corresponding side of the other Δ is 5.4 cm

36. Triangles ABC and DEF are similar. If $ar(\Delta ABC) = 9\ cm^2,\ ar(\Delta DEF) = 64\ cm^2$ $and\ DE = 5.1\ cm$, find AB.

Ans $\because \Delta ABC \sim \Delta DEF$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} \qquad \left[\begin{array}{c}\because the\ ratio\ of\ two\ similar\ \Delta s\ is\ equal\ to\ the\ ratio \\ of\ their\ corresponding\ sides\end{array}\right]$$

$$= \sqrt{\frac{9}{64}} = \sqrt{\frac{AB^2}{(5.1cm)^2}} \qquad [Taking\ square\ root\ of\ both\ sides]$$

$$\Rightarrow \frac{3}{8} = \frac{AB}{5.1\ cm}$$

$\Rightarrow \; 8\ \times AB = 3 \times 5.1\ cm$

$$\Rightarrow \; AB = \frac{15.3}{8}\ cm$$

$\Rightarrow \; AB = 1.91\ cm$

37. The side of a certain triangle are given as $a = 3\ cm, b = 4\ cm\ and\ c = 7\ cm$. Determine whether it is a right angled triangle or not.

Ans Here,

$a = 3\ cm, b = 4\ cm\ and\ c = 7\ cm$

$\therefore\ a^2 = (3\ cm)^2 = 9\ cm^2$

$b^2 = (4\ cm)^2 = 16\ cm^2$

$c^2 = (7\ cm)^2 = 49\ cm^2$

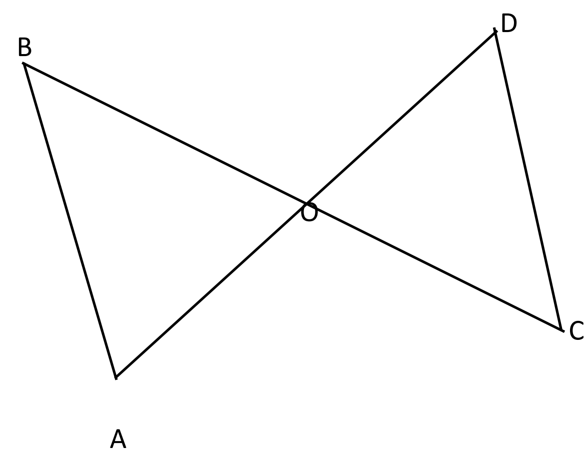
$\therefore\ a^2 + b^2 = 9\ cm^2 + 16\ cm^2$

$= 25\ cm^2$

$\neq c^2$

$\therefore$ Hence, the triangle is not a right angled triangle.

38. In the adjoining figure, $\Delta OAB \sim \Delta OCD$. When $AB = 8cm, BO = 6.4cm, OC = 3.5cm\ and\ CD = 5cm$, find $OA\ and\ DO$.



Ans $\because \Delta OAB \sim \Delta OCD$

$\therefore \frac{AB}{CD} = \frac{OA}{OC} = \frac{OB}{OD}$

$\Rightarrow \frac{8}{5} = \frac{OA}{3.5\ cm} = \frac{6.4\ cm}{OD}$

Taking, $\frac{8}{5} = \frac{OA}{3.5\ cm}$

$\Rightarrow 5 \times OA = 8 \times 3.5\ cm$

$\Rightarrow OA = 5.6\ cm$

Taking, $\frac{8}{5} = \frac{6.4\ cm}{OD}$

$\Rightarrow 8 \times OD = 5 \times 6.4\ cm$

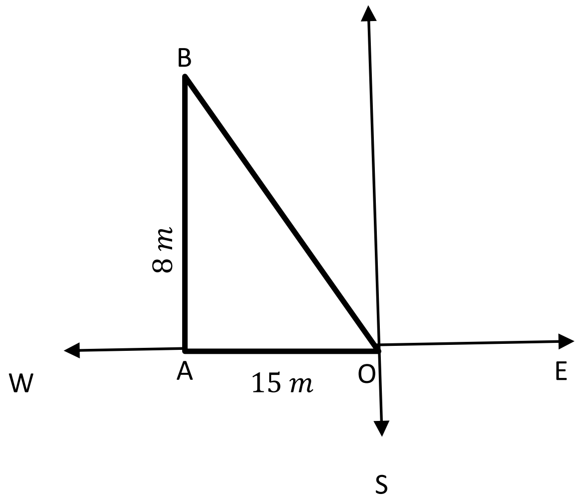
$\Rightarrow OD = 4\ cm$

39. A man goes 15 m due west and then 8 m due north. How far is he from the startin point?

Ans Let O be the starting point

$OA = 15m\ and\ AB = 8m$

In right ΔOAB



$$OB^2 = OA^2 + AB^2 \quad [By\ Pythagoras\ Theorem]$$

$$\Rightarrow OB^2 = (15m)^2 + (8m)^2$$

$$\Rightarrow OB^2 = 225m^2 + 64m^2$$

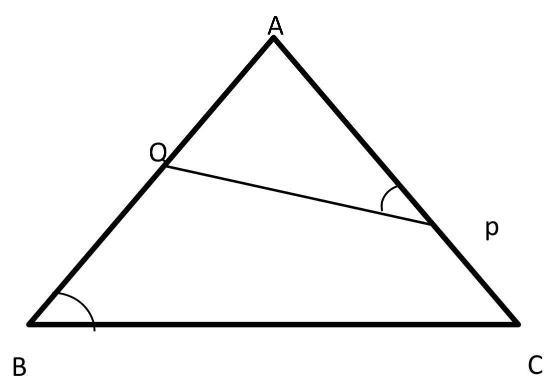
$$\Rightarrow OB^2 = 289m^2$$

$$\Rightarrow \text{OB} = \sqrt{289m^2}$$

$$\Rightarrow \text{OB} = 17m$$

$\therefore$ The distance of the man from starting point is 17m

40. In the adjoining figure, $\angle APQ = \angle B$. Prove that $\Delta APQ \sim \Delta ABC$. If $AP = 3.8cm, AQ = 3.6cm, BQ = 2.1cm\ and\ BC = 4.2cm\ find\ PQ$.



Ans In $\Delta APQ\ and\ \Delta ABC$,

$\angle A = \angle A \quad (Common\ angle)$

$\angle APQ = \angle B \quad (Given)$

$\therefore \Delta APQ \sim \Delta ABC \quad (AA\ Similarity)$

$$\therefore \frac{AP}{AB} = \frac{PQ}{BC}$$

$$\Rightarrow \frac{3.8}{3.6 + 2.1} = \frac{PQ}{4.2}$$

$$\Rightarrow \frac{3.8}{5.7} = \frac{PQ}{4.2}$$

$$\Rightarrow 5.7 \times PQ = 3.8 \times 4.2\ cm$$

$$\Rightarrow \frac{57}{10} \times PQ = \frac{38}{10} \times \frac{42}{10}\ \text{cm}$$

$$\Rightarrow PQ = \frac{14}{5}\ \text{cm}$$

$$\Rightarrow PQ = 2.8\ cm$$

Chapter 8: Introduction to Trigonometry

41. If $Cos\theta = \frac{3}{5}$, find the value of $sin\theta\ and\ cot\theta$.

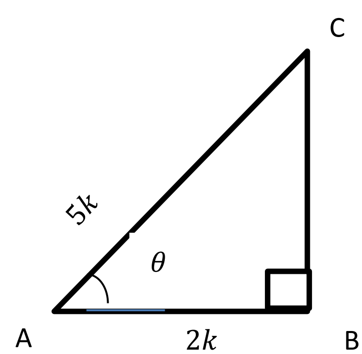
Ans ABC is a right angled Δ, right angled at B

We have $Cos\theta = \frac{Base}{Hypotenuse}$

$$= \frac{AB}{AC}$$

$$= \frac{3}{5}$$

By Pythagoras Theorem in ΔABC, we get



$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (5k)^2 = (3k)^2 + (BC)^2$$

$$\Rightarrow 25k^2 = 9k^2 + BC^2$$

$$\Rightarrow BC^2 = 25k^2 - 9k^2$$

$$\Rightarrow BC^2 = 16k^2$$

$$\Rightarrow AC^2 = \sqrt{16k^2}$$

$\Rightarrow\ AC = 4k$

$\therefore\ \ sin\theta = \frac{Perpendicular}{Hypotenuse}$

$= \frac{BC}{AC}$

$= \frac{4k}{5k}$

$= \frac{4}{5}$

$cot\theta = \frac{Base}{Perpendicular}$

$= \frac{AB}{BC}$

$= \frac{3k}{4k}$

$= \frac{3}{4}$

42. Find the value of $Sin60^0 Cos30^0 + Cos60^0 Sin30^0$

Ans $Sin60^0 Cos30^0 + Cos60^0 Sin30^0$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4}$

$= \frac{3+1}{4}$

$= \frac{4}{4}$

$= 1$

43. If $A = 30^0$, verify that $sin2A = 2sinA\ cosA$

Ans L H S $= sin2A$

$= sin2 \times 30^0$

$= sin60^0$

$= \frac{\sqrt{3}}{2}$

R H S $= 2sinA\ cosA$

$= 2sin30^0 \times cos30^0$

$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

$\therefore$ L H S = R H S

Hence Verified

44. If $\sin(A + B) = 1$ *and* $\cos(A - B) = 1, 0^0 < A + B \leq 90^0, A >$, find A *and* B.

Ans $\sin(A + B) = 1$

$\Rightarrow \sin(A + B) = sin90^0$

$\Rightarrow A + B = 90^0$

$\Rightarrow A = 90^0 - B$ -------------(i)

$\cos(A - B) = 1$

$\Rightarrow \cos(A - B) = cos0^0$

$\Rightarrow A - B = 0^0$

$\Rightarrow 90^0 - B - B = 0^0$ $[using\ eqn\ (i)]$

$\Rightarrow -2B = -90^0$

$\Rightarrow \text{B} = \frac{90^0}{2}$

$\Rightarrow \text{B} = 45^0$

Putting $\text{B} = 45^0$ in eqn (i), we get

$\text{A} = 90^0 - B$

$= 90^0 - 45^0$

$\therefore \text{A} = 45^0$

$\text{B} = 45^0$

45. Evaluate: $\left(\frac{\sin 25^0}{\cos 65^0}\right)^2 - \left(\frac{\cos 61^0}{\sin 29^0}\right)^2$

Ans $\left(\frac{\sin 25^0}{\cos 65^0}\right)^2 - \left(\frac{\cos 61^0}{\sin 29^0}\right)^2$

$= \left\{\frac{\sin 25^0}{\sin(90^0-65^0)}\right\}^2 - \left\{\frac{\cos 25^0}{\cos(90^0-29^0)}\right\}^2$

$= \left\{\frac{\sin 25^0}{\sin 25^0}\right\}^2 - \left\{\frac{\cos 25^0}{\cos 25^0}\right\}^2$

$= 1^2 - 1^2$

$= 1 - 1$

$= 0$

46. Prove that $\frac{\sin 10^0}{\cos 80^0} + \cos 57^0 \, cosec \, 33^0 = 1$

Ans L H S $= \frac{\sin 10^0}{\cos 80^0} + \cos 57^0 \, cosec \, 33^0$

$= \frac{\cos(90^0-10^0)}{\cos 80^0} + \sin(90^0 - 57^0) \, cosec \, 33^0$

$= \frac{\cos 80^0}{\cos 80^0} + \sin 33^0 \, cosec \, 33^0$

$= 1 + \sin 33^0 \times \frac{1}{\sin 33^0}$

$= 1 + 1$

$= 2$

= R H S

Hence Proved

47. Prove that $(1 - Sin^2\theta)Sec^2\theta = 1$

Ans L H S $= (1 - Sin^2\theta)Sec^2\theta$

$= Cos^2\theta \times Sec^2\theta$

$= Cos^2\theta \times \frac{1}{Cos^2\theta}$

$= 1$

= R H S

Hence Proved

48. Prove that $\frac{cos\theta}{1-sin\theta} = \frac{1+sin\theta}{cos\theta}$

Ans LHS $= \frac{cos\theta}{1-sin\theta}$

$= \frac{cos\theta\ (1+sin\theta)}{(1-sin\theta)(1+sin\theta)}$

$= \frac{cos\theta\ (1+sin\theta)}{1^2-sin^2\theta}$

$= \frac{cos\theta\ (1+sin\theta)}{1-sin^2\theta}$

$= \frac{cos\theta\ (1+sin\theta)}{cos^2\theta}$

$= \frac{1 + sin\theta}{cos\theta}$

= RHS

Hence Proved

49. If $x\ cosec\theta = a\ and\ y\ cot\theta = b$ prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$

Ans L H S $= \frac{a^2}{x^2} - \frac{b^2}{y^2}$

$= \frac{(x\ cosec\theta)^2}{x^2} - \frac{(y\ cot\theta)^2}{y^2}$

$= \frac{x^2 cosec^2\theta}{x^2} - \frac{y^2\ cot^2\theta}{y^2}$

$= cosec^2\theta - cot^2\theta$

$= 1$

= R H S

Hence Proved

50. Prove that $(Sec^2A - 1)(Cosec^2A - 1) = 1$

Ans L H S $= (Sec^2A - 1)(Cosec^2A - 1)$

$= \ tan^2A \times cot^2A$

$= \ tan^2A \times \frac{1}{tan^2A}$

$= 1$

= R H S

Hence Proved

Chapter 14: Probability

51. A die is thrown once. What is the probability of getting a number less than 3?

Ans Total number of possible outcomes $\{ie\ 1, 2, 3, 4, 5, 6\} = 6$

Number of favourable outcomes less than 3 $\{ie\ 1, 2\} = 2$

$\therefore$ P (less than 3) $= \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ possible\ outcomes}$

$= \frac{2}{6}$

$= \frac{1}{3}$

52. A bag contains 6 red balls, 8 white balls, 5 green balls and 3 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is red or black.

Ans Total number of balls in the bag = 6 + 8 + 5 + 3

= 22

$\therefore$ Total number of possible outcomes = 22

Number of favourable outcomes = 6 + 3 = 9

$\therefore$ P (red or black) $= \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ possible\ outcomes}$

$= \frac{9}{22}$

53. A box contains 20 cards number 1 to 20. A card is drawn at random from the box. Find the probability that the number on the drawn card is a prime number.

Ans Total number of possible outcomes = 20

Prime numbers are 2, 3, 5, 7, 11, 13, 17, 19

$\therefore$ Number of favourable outcomes of a prime number = 8

$\therefore$ P (prime number) $= \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ possible\ outcomes}$

$= \frac{8}{20}$

$= \frac{2}{5}$

54. One card is drawn at random from a well shuffled deck of 52 cards. Find the probability that the card drawn is
    (i) A black card
    (ii) A spade

Ans Total number of possible outcomes = 52

(i) Number of black cards = 26

$\therefore$ Number of favourable outcomes = 26

$\therefore$ P (a black card) $= \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ possible\ outcomes}$

$= \frac{26}{52}$

$= \frac{1}{2}$

(ii) Number of spade = 13

$\therefore$ Number of favourable outcomes = 13

$\therefore$ P (a spade) $= \frac{13}{52}$

$= \frac{1}{4}$

55. A bag contains lemon flavoured candies only. Maline takes out one candy without looking into the bag. What is the probability that she takes out an orange flavoured candy.

Ans Let the number of lemon flavoured candies be x

$\therefore$ Total number of favourable outcomes $= x$

Number or orange flavoured candies $= 0$

$\therefore$ P (orange flavoured candies) $= \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ possible\ outcomes}$

$= \frac{0}{x}$

$= 0$

56. Cards marked with numbers 13, 14, 15, . . ., 60 are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that the number on the drawn card is divisible by 5.

Ans Number are 13, 14, 15, . . ., 60

$\therefore$ Total number of possible outcomes = 60 – 12 = 48

Numbers divisible by 5 between 13 to 60 are

15, 20, 25, 30, 35, 40, 45, 50, 55, 60

$\therefore$ Number of favourable outcomes = 10

$\therefore$ P (divisible by 5) $= \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ possible\ outcomes}$

$= \frac{10}{48}$

$= \frac{5}{24}$

57. Cards bearing numbers 1, 3, 5, . . ., 35 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing a prime number less than 15.

Ans Total number of possible outcomes $= \frac{35+1}{2} = \frac{36}{2} = 18$

Cards bearing a prime number less than 15 are 3, 5, 7, 11, 13

$\therefore$ Number of favourable outcomes = 5

$\therefore$ P (a prime number less than 15) $= \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ possible\ outcomes}$

$= \frac{5}{18}$

58. A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball is twice that of the red ball, find the number of blue balls in the bag.

Ans Number of red balls = 6

Let the number of blue balls be x

$\therefore$ The number of balls $= 6 + x$

$\therefore$ The number of possible outcomes $= 6 + x$

Given,

P (blue ball) = 2 × P (red ball)

$\Rightarrow \frac{x}{6+x} = 2 \times \frac{6}{6+x}$

$\Rightarrow x(6 + x) = 12(6 + x)$

$\Rightarrow x = \frac{12\ (6+x)}{6+x}$

$\Rightarrow\ x = 12$

$\therefore$ Number of blue balls is 12

59. There are 30 cards of the same size, in a bag on which numbers 1 to 30 are written. One card is drawn out of the bag at random. Find the probability that the number on the selected card is not divisible by 3.

Ans Total number of possible outcomes = 30

Numbers divisible by 3 are 3, 6, 9, 12, 15, 18, 24, 27, 30

$\therefore$ Numbers of favourable outcomes of divisible by 3 is 10

$\therefore$ P (not divisible by3)

$= 1 - P(divisible\ by\ 3)$

$= 1 - \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ possible\ outcomes}$

$= 1 - \frac{10}{30}$

$= 1 - \frac{1}{3}$

$= \frac{3-1}{3}$

$= \frac{2}{3}$

60. The king, queen, jack and 10, all of spades are lost from the pack of 52 playing cards. A card is drawn from the remaining well-shuffled pack. Find the probability of getting a
    i) Red card
    ii) King

Ans Total number of possible outcomes = 52 – 4 = 48

i) Number of red cards = 26

$\therefore$ Number of favourable outcomes = 26

$$\therefore \text{ P (red card)} = \frac{\textit{number of favourable outcomes}}{\textit{total number of possible outcomes}}$$

$$= \frac{26}{48}$$

$$= \frac{13}{24}$$

ii) Number of keys = 3

$\therefore$ Number of favourable outcomes = 3

$$\therefore \text{ P (king)} = \frac{3}{48}$$

$$= \frac{1}{16}$$

Section-C
Short Answer Questions (3 Marks)

[Chapter-2: Polynomials]

1. Find the zeroes of the quadratic polynomial $2x^2 + x - 10$ and verify the relationship between the zeroes and the coefficients.

Soln.: Let $p(x) = 2x^2 + x - 10$

$= 2x^2 + 5x - 4x - 10$

$= x(2x + 5) - 2(2x + 5)$

$= (2x + 5)(x - 2)$

Now, $p(x) = 0$

⇨ $(2x + 5)(x - 2) = 0$

Either, $2x + 5 = 0$ $or,$ $x - 2 = 0$

⇨ $x = -\frac{5}{2}$ $or,$ $x = 2$

Hence, the zeroes of $p(x)$ are $-\frac{5}{2}$ and 2.

Verification:

Sum of the zeroes $= -\frac{5}{2} + 2$

$= \frac{-5+4}{2}$

$= -\frac{1}{2}$

$= -\frac{coefficient\ of\ x}{coefficient\ of x^2}$

Product of zeroes $= -\frac{5}{2} \times 2$

$= -\frac{10}{2}$

$= \frac{constant\ term}{coefficient\ of\ x^2}$

Hence, verified.

2. Find a quadratic polynomial whose zeroes are $3 + \sqrt{5}$ and $3 - \sqrt{5}$.

Soln.: Sum of the zeroes $= 3 + \sqrt{5} + 3 - \sqrt{5}$

$= 6$

Product of zeroes $= (3 + \sqrt{5})(3 - \sqrt{5})$

$= (3)^2 - (\sqrt{5})^2$

$= 9 - 5$

$= 4$

Therefore, the required quadratic polynomial

$= x^2 - (sum\ of\ zeroes)x + Product\ of\ zeros$

$= x^2 - 6x + 4.$ (Ans)

3. Check whether $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$ by using division algorithm.

Soln.: We have,

$x^2 + 3x + 1)3x^4 + 5x^3 - 7x^2 + 2x + 2(3x^2 - 4x + 2$

$3x^2 + 9x^3 + 3x^2$

$(-) \quad (-) \quad (-)$

$-4x^3 - 10x^2 + 2x + 2$

$-4x^3 - 12x^2 - 4x$

$(+) \quad (+) \quad (+)$

$2x^2 + 6x + 2$

$\mathbf{2x^2 + 6x + 2}$

$(-) \quad (-) \quad (-)$

$\mathbf{0}$

Since, the remainder is $\mathbf{0}$.

Therefore, $\boldsymbol{x^2 + 3x + 1}$ is a factor of $\boldsymbol{x^4 + 5x^3 - 7x^2 + 2x + 2}$. (Ans)

4. If one zero of the polynomial $\boldsymbol{(a^2 + 9)x^2 + 15x + 6a}$ is a reciprocal of the other, then find the value of $\boldsymbol{a}$.

Soln.: Given polynomial is $\boldsymbol{(a^2 + 9)x^2 + 15x + 6a}$

Let the other zero be $\boldsymbol{y}$

$\therefore$ One zero $= \frac{1}{y}$

Now,

Product of the zeroes $= \frac{Constant\ term}{Coefficient\ of\ x^2}$

⇨ $\boldsymbol{y \times \frac{1}{y} = \frac{6a}{a^2+9}}$

⇨ $\boldsymbol{1 = \frac{6a}{a^2+9}}$

⇨ $\boldsymbol{a^2 + 9 = 6a}$

⇨ $\boldsymbol{a^2 - 6a + 9 = 0}$

⇨ $\boldsymbol{a^2 - (3 + 3)a + 9 = 0}$

⇨ $\boldsymbol{a^2 - 3a - 3a + 9 = 0}$

⇨ $\boldsymbol{a(a - 3) - 3(a - 3) = 0}$

⇨ $\boldsymbol{(a - 3)(a - 3) = 0}$

Either, $\boldsymbol{a - 3 = 0}$ *or*, $\boldsymbol{a - 3 = 0}$

⇨ $\boldsymbol{a = 3}$ *or*, $\boldsymbol{a = 3}$

Hence, the value of $\boldsymbol{a}$ is $\mathbf{3}$. (Ans)

5. If $\boldsymbol{\alpha}$ ***and*** β are the zeroes of $\boldsymbol{x^2 - 3x + 1}$, then find a polynomial whose zeroes are $\frac{\alpha}{\beta}$ ***and*** $\frac{\beta}{\alpha}$.

Soln.: Let $\boldsymbol{p(x) = x^2 - 3x + 1}$

Since, $\boldsymbol{\alpha}$ ***and*** $\boldsymbol{\beta}$ are the zeroes of $\boldsymbol{p(x)}$.

∴Sum of the zeroes $= -\frac{coefficient\ of\ x}{coefficient\ of\ x^2}$

⇨ $\alpha + \beta = -\frac{-3}{1}$

⇨ $\alpha + \beta = 3$

& Product of the zeroes $= \frac{constant\ term}{coefficient\ of\ x^2}$

⇨ $\alpha\beta = \frac{1}{1}$

⇨ $\alpha\beta = 1$

Now, a quadratic polynomial whose zeroes are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

$= x^2 - (Sum\ of\ the\ zeroes)x + Product\ of\ the\ zeroes$

$= x^2 - \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \frac{\alpha}{\beta} \times \frac{\beta}{\alpha}$

$= x^2 - \left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)x + 1$

$= x^2 - \left\{\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\right\}x + 1$

$= x^2 - \left\{\frac{(3)^2-2\times1}{1}\right\}x + 1$

$= x^2 - 7x + 1$ (Ans)

6. If two zeroes of the polynomial $f(x) = x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then find its third zero.

Soln.: Given: $f(x) = x^3 - 4x^2 - 3x + 12$

$= x^3 - 3x - 4x^2 + 12$

$= x(x^2 - 3) - 4(x^2 - 3)$

$= (x^2 - 3)(x - 4)$

To find the zeroes of the polynomial, we put

$f(x) = 0$

⇨ $(x^2 - 3)(x - 4) = 0$

Either, $(x^2 - 3) = 0$ *or*, $(x - 4) = 0$

⇨ $x = \pm\sqrt{3}$ *or*, $x = 4$

⇨ $x = \sqrt{3}, -\sqrt{3}$

Hence, the third zero is **4**. (Ans)

7. If one zero of the polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Soln.: Let one zero of $p(x) = 3x^2 - 8x + 2k + 1$ be y.

∴ The other zero $= 7y$.

So,

Sum of the zeroes $= -\frac{coefficient\ of\ x}{coefficient\ of\ x^2}$

⇨ $y + 7y = -\frac{(-8)}{3}$

⇨ $8y = \frac{8}{3}$

⇨ $y = \frac{1}{3}$ -----------------(i)

And,

Product of the zeroes = $\frac{constant\ term}{coefficient\ of\ x^2}$

⇨ $y \times 7y = \frac{2k+1}{3}$

⇨ $7y^2 = \frac{2k+1}{3}$

⇨ $7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$ $[from\ (i)]$

⇨ $\frac{7}{9} = \frac{2k+1}{3}$

⇨ $9(2k+1) = 7 \times 3$

⇨ $18k + 9 = 21$

⇨ $18k = 21 - 9$

⇨ $k = \frac{12}{18}$

⇨ $k = \frac{2}{3}$ (Ans)

8. On dividing $3x^3 + x^2 + 2x + 5$ by a polynomial $g(x)$, the quotient and remainder are $(3x - 5)$ and $(9x + 10)$ respectively. Find $g(x)$.

Soln.: Here, Dividend = $3x^3 + x^2 + 2x + 5$

Quotient = $3x - 5$

And Remainder = $9x + 10$

Divisor = $g(x)$

By Division algorithm,

$Dividend = Divisor \times Quotient + Remainder$

⇨ $3x^3 + x^2 + 2x + 5 = g(x) \times (3x - 5) + (9x + 10)$

⇨ $3x^3 + x^2 + 2x + 5 - 9x - 10 = g(x) \times (3x - 5)$

⇨ $3x^3 + x^2 - 7x - 5 = g(x) \times (3x - 5)$

⇨ $g(x) = \frac{3x^3 + x^2 - 7x - 5}{3x - 5}$ -----------------(i)

So, Dividing $3x^3 + x^2 - 7x - 5$ by $3x - 5$, $we\ get$

$$
\begin{array}{r}
3x-5\overline{)3x^3+x^2-7x-5}(x^2+2x+1 \\
3x^3-5x^2 \qquad\qquad \\
(-) \quad (+) \qquad\qquad \\
\hline
6x^2-7x \qquad \\
6x^2-10x \qquad \\
(-) \quad (+) \qquad \\
\hline
3x-5 \\
3x-5 \\
(-) \quad (+) \\
\hline
0
\end{array}
$$

∴ From (i), we have

$$g(x) = \frac{3x^3+x^2-7x-5}{3x-5}$$

$$= \frac{(3x-5)(x^2+2x+1)}{(3x-5)}$$

$$= x^2+2x+1$$

Hence, $g(x) = x^2+2x+1$. (Ans)

9. If α *and* β are the zeroes of the polynomial $p(x) = 3x^2-2x-6$, then find $\alpha^3+\beta^3$.

Soln.: Given : $p(x) = 3x^2-2x-6$

Sum of the zeroes $= -\frac{coefficient\ of\ x}{coefficient\ of\ x^2}$

⇨ $\alpha + \beta = \frac{-(-2)}{3}$

⇨ $\alpha + \beta = \frac{2}{3}$

Product of the zeroes $= \frac{constant\ term}{coefficient\ of\ x^2}$

⇨ $\alpha \times \beta = \frac{-6}{3}$

⇨ $\alpha \times \beta = -2$

Now,

$$\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\,\alpha\beta(\alpha+\beta)$$

$$= \left(\frac{2}{3}\right)^3 - 3(-2)\left(\frac{2}{3}\right)$$

$$= \frac{8}{27} + 4$$

$$= \frac{8+108}{27}$$

$$= \frac{116}{27} \quad \text{(Ans)}$$

10. Divide $6x^3+11x^2-39x-65$ by x^2+x-1 and hence verify division algorithm for the polynomials.

Soln.: Dividend $= 6x^3+11x^2-39x-65$

Divisor $= x^2+x-1$

So,

$$
\begin{array}{r}
x^2+x-1\ )\,6x^3+11x^2-39x-65\ (6x+5 \\
6x^3+6x^2-6x \qquad\qquad \\
(-)\quad(-)\quad(+) \qquad\qquad \\
\hline
5x^2-33x-65 \\
5x^2+5x-5 \\
(-)\quad(-)\quad(+) \\
\hline
-38x-60
\end{array}
$$

∴ Quotient $= \boldsymbol{6x+5}$

Remainder $= \boldsymbol{-38x-60}$

Verification:

$\boldsymbol{Divisor \times Quotient + Remainder}$

$= (\boldsymbol{x^2+x-1}) \times (\boldsymbol{6x+5}) + (\boldsymbol{-38x-60})$

$= \boldsymbol{6x^3+6x^2-6x+5x^2+5x-5-38x-60}$

$= \boldsymbol{6x^3+11x^2-39x-65}$

$= \boldsymbol{Dividend}$

Hence, Verified.

CHAPTER – 4

QUADRATIC EQUATIONS

11. Solve the equation $\boldsymbol{10x-\frac{1}{x}=3}$ by factorisation.

Soln.: $\boldsymbol{10x-\frac{1}{x}=3}$

⇨ $\boldsymbol{\frac{10x^2-1}{x}=3}$

⇨ $\boldsymbol{10x^2-1=3x}$

⇨ $\boldsymbol{10x^2-3x-1=0}$

⇨ $\boldsymbol{10x^2-(5-2)x-1=0}$

⇨ $\boldsymbol{10x^2-5x+2x-1=0}$

⇨ $\boldsymbol{5x(2x-1)+1(2x-1)=0}$

⇨ $\boldsymbol{(2x-1)(5x+1)=0}$

Either, $\boldsymbol{2x-1=0}$ ***or***, $\boldsymbol{5x+1=0}$

⇨ $\boldsymbol{x=\frac{1}{2}}$ ***or***, $\boldsymbol{x=-\frac{1}{5}}$

Hence, $\boldsymbol{x=\frac{1}{2}}$ ***and*** $\boldsymbol{x=-\frac{1}{5}}$ are the roots of the given equation. (Ans)

12. Solve the quadratic equation $\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$ by completing the squares.

<u>Soln.:</u> The given equation is –

$$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$$

$\Rightarrow \frac{\sqrt{2}x^2}{\sqrt{2}} - \frac{3}{\sqrt{2}}x - \frac{2\sqrt{2}}{\sqrt{2}} = 0$ [Dividing throughout by $\sqrt{2}$]

$\Rightarrow x^2 - \frac{3}{\sqrt{2}}x - 2 = 0$

$\Rightarrow x^2 - \frac{3}{\sqrt{2}}x = 2$

$\Rightarrow x^2 - 2 \times x \times \frac{3}{2\sqrt{2}} + \left(\frac{3}{2\sqrt{2}}\right)^2 = 2 + \left(\frac{3}{2\sqrt{2}}\right)^2$

$\Rightarrow \left(x - \frac{3}{2\sqrt{2}}\right)^2 = 2 + \frac{9}{8}$

$\Rightarrow \left(x - \frac{3}{2\sqrt{2}}\right)^2 = \frac{16+9}{8}$

$\Rightarrow x - \frac{3}{2\sqrt{2}} = \pm\sqrt{\frac{25}{8}}$

$\Rightarrow x - \frac{3}{2\sqrt{2}} = \pm\frac{5}{2\sqrt{2}}$

$\Rightarrow x = \frac{3}{2\sqrt{2}} \pm \frac{5}{2\sqrt{2}}$

$\Rightarrow x = \frac{3\pm5}{2\sqrt{2}}$

$\Rightarrow x = \frac{3+5}{2\sqrt{2}}\ ,\ \frac{3-5}{2\sqrt{2}}$

$\Rightarrow x = \frac{8}{2\sqrt{2}}\ ,\ \frac{-2}{2\sqrt{2}}$

$\Rightarrow x = \frac{4}{\sqrt{2}}\ , -\frac{1}{\sqrt{2}}$

$\Rightarrow x = 2\sqrt{2}\ , -\frac{\sqrt{2}}{2}$

Hence, $x = 2\sqrt{2}$ *and* $x = \frac{-\sqrt{2}}{2}$ are the roots of the given equation. (Ans)

13. Examine whether the quadratic equation $2x^2 + x - 6 = 0$ have real roots. If so, find the roots.

<u>Soln.:</u> Quadratic equation is $2x^2 + x - 6 = 0$.

Comparing it with $ax^2 + bx + c = 0$, we have

$a = 2,\ b = 1,\ c = -6$

Now,

Discriminant, D $= b^2 - 4ac$

$= (1)^2 - 4 \times 2 \times (-6)$

$= 1 + 48$

$= 49$

Since, $D > 0$

∴ The given equation has real roots given by –

$$x = \frac{-b\pm\sqrt{D}}{2a}$$

⇨ $x = \frac{-1\pm\sqrt{49}}{2\times2}$

⇨ $x = \frac{-1\pm7}{4}$

⇨ $x = \frac{-1+7}{4}$, $\frac{-1-7}{4}$

⇨ $x = \frac{6}{4}$, $-\frac{8}{4}$

⇨ $x = \frac{3}{2}$, -2.

Hence, $\frac{3}{2}$ and -2 are the roots of the given equation. (Ans)

14. Find the value of k for which the quadratic equation $2kx^2 - 40x + 25 = 0$ have real and equal roots. Also, find the roots.

Soln.: The given equation is –

$2kx^2 - 40x + 25 = 0$ --------------------(i)

Comparing it with $ax^2 + bx + c = 0$, we have

$a = 2k$, $b = -40$, $c = 25$

So, Discriminant, D $= b^2 - 4ac$

$= (-40)^2 - 4 \times 2k \times 25$

$= 1600 - 200k$

For real and equal roots,

Discriminant, D $= 0$

⇨ $1600 - 200k = 0$

⇨ $1600 = 200k$

⇨ $k = \frac{1600}{200}$

⇨ $k = 8$.

Putting $k = 8$ in eqn. (i), we get

$2kx^2 - 40x + 25 = 0$

⇨ $2 \times 8 \times x^2 - 40x + 25 = 0$

⇨ $16x^2 - 40x + 25 = 0$

⇨ $(4x)^2 - 2 \times 4x \times 5 + (5)^2 = 0$

⇨ $(4x - 5)^2 = 0$

⇨ $(4x - 5)(4x - 5) = 0$

⇨ $x = \frac{5}{4}$, $\frac{5}{4}$

Hence, the real and equal roots of the given quadratic equation are $\frac{5}{4}$. (Ans)

15. The sum of two numbers is 18. The sum of their reciprocals is $\frac{1}{4}$. Find the numbers.

Soln.: Given, sum of two numbers is 18.

Let the required numbers be x and $(18 - x)$.

Then, By the question, we have

$$\frac{1}{x} + \frac{1}{18-x} = \frac{1}{4}$$

⇨ $\frac{18-x+x}{x(18-x)} = \frac{1}{4}$

⇨ $\frac{18}{18x-x^2} = \frac{1}{4}$

⇨ $\mathbf{72 = 18x - x^2}$

⇨ $\mathbf{x^2 - 18x + 72 = 0}$

⇨ $\mathbf{x^2 - 12x - 6x + 72 = 0}$

⇨ $\mathbf{x(x - 12) - 6(x - 12) = 0}$

⇨ $\mathbf{(x - 12)(x - 6) = 0}$

Either, $\mathbf{x - 12 = 0}$ $\mathbf{or}$, $\mathbf{x - 6 = 0}$

⇨ $\mathbf{x = 12}$ $\mathbf{or}$, $\mathbf{x = 6.}$

Hence, the two numbers are $\mathbf{6\ and\ 12.}$ (Ans)

16. Divide 16m into two parts such that twice the square of the greater part exceeds the square of the smaller part by 164.

Soln.: Let the greater part be $\mathbf{x}$.

∴ Smaller part will be $(\mathbf{16 - x})$.

Now,

According to the question, we have

$$\mathbf{2x^2 = (16 - x)^2 + 164}$$

⇨ $\mathbf{2x^2 = (16)^2 - 2 \times 16x + x^2 + 164}$

⇨ $\mathbf{2x^2 = 256 - 32x + x^2 + 164}$

⇨ $\mathbf{2x^2 - x^2 + 32x - 256 - 164 = 0}$

⇨ $\mathbf{x^2 + 32x - 420 = 0}$

⇨ $\mathbf{x^2 + 42x - 10x - 420 = 0}$

⇨ $\mathbf{x(x + 42) - 10(x + 42) = 0}$

⇨ $\mathbf{(x + 42)(x - 10) = 0}$

Either, $\mathbf{x + 42 = 0}$ $\mathbf{or}$, $\mathbf{x - 10 = 0}$

⇨ $\mathbf{x = -42}$ $\mathbf{or}$, $\mathbf{x = 10}$

But, $\mathbf{x = -42}$ is rejected as distance cannot be negative.

Hence, two parts are $\mathbf{10\ m\ and\ (16 - 10) = 6\ m.}$ (Ans)

17. The sum of the squares of two consecutive natural numbers is 421. Find the numbers.

Soln.: Let the two consecutive natural numbers be $\mathbf{x\ and\ (x + 1)}$.

Now, According to the question, we have

$$\mathbf{x^2 + (x + 1)^2 = 421}$$

⇨ $\mathbf{x^2 + x^2 + 2.x.1 + 1^2 = 421}$

⇨ $\mathbf{2x^2 + 2x + 1 - 421 = 0}$

⇨ $\mathbf{2x^2 + 2x - 420 = 0}$

⇨ $\mathbf{2(x^2 + x - 210) = 0}$

⇨ $x^2 + x - 210 = 0 \;\; or, \;\; 2 \neq 0$
⇨ $x^2 + 15x - 14x - 210 = 0$
⇨ $x(x + 15) - 14(x + 15) = 0$
⇨ $(x + 15)(x - 14) = 0$
Either, $x + 15 = 0 \quad or, \quad x - 14 = 0$
⇨ $x = -15 \quad or, \quad x = 14$
As -15 is not a natural number, so it is rejected.
Hence, the required two consecutive natural numbers are 14 *and* $(14 + 1) = 15$. (Ans)

18. Solve: $\frac{3}{x+1} - \frac{1}{2} = \frac{2}{3x-1}$, where $x \neq -1, \frac{1}{3}$.

Soln.: $$\frac{3}{x+1} - \frac{1}{2} = \frac{2}{3x-1}$$
⇨ $\frac{6-1(x+1)}{2(x+1)} = \frac{2}{3x-1}$
⇨ $\frac{6-x-1}{2x+2} = \frac{2}{3x-1}$
⇨ $\frac{5-x}{2x+2} = \frac{2}{3x-1}$
⇨ $2(2x + 2) = (5 - x)(3x - 1)$
⇨ $4x + 4 = 15x - 5 - 3x^2 + 1x$
⇨ $3x^2 + 4x - 16x + 4 + 5 = 0$
⇨ $3x^2 - 12x + 9 = 0$
⇨ $3(x^2 - 4x + 3) = 0$
⇨ $x^2 - 4x + 3 = 0 \;\; or, \;\; 3 \neq 0$
⇨ $x^2 - 3x - x + 3 = 0$
⇨ $x(x - 3) - 1(x - 3) = 0$
⇨ $(x - 3)(x - 1) = 0$
Either, $x - 3 = 0 \quad or, \quad x - 1 = 0$
⇨ $x = 3 \quad or, \quad x = 1$
Hence, $x = 1, \; 3$. (Ans)

19. A girl is twice as old as her sister. Four years hence the product of their ages (in years) will be 160. Find their present ages.

Soln.: Let the sister's present age be x years.
∴ Girls present age $= 2x$ years.
Four years hence, Sister's age $= (x + 4)$ years
Girl's age $= (2x + 4)$ years
Now, According to the question, we have
$$(x + 4)(2x + 4) = 160$$
⇨ $2x^2 + 8x + 4x + 16 = 160$
⇨ $2x^2 + 12x + 16 - 160 = 0$
⇨ $2x^2 + 12x - 144 = 0$

⇨ $2(x^2 + 6x - 72) = 0$

⇨ $x^2 + 6x - 72 = 0$ $or,$ $2 \neq 0$

⇨ $x^2 + 12x - 6x - 72 = 0$

⇨ $x(x + 12) - 6(x + 12) = 0$

⇨ $(x + 12)(x - 6) = 0$

Either, $x + 12 = 0$ $or,$ $x - 6 = 0$

⇨ $x = -12$ $or,$ $x = 6$

Here, $x = -12$ is rejected as age cannot be negative.

$\therefore x = 6$ years

Hence, the present age of sister $= 6$ years.

& The present age of a girl$= 2 \times 6 = 12$ years. (Ans)

20. Find the value of k for which the equation $x^2 - 4x + k = 0$ has distinct real roots.

Soln.: The given equation is $x^2 - 4x + k = 0$

Comparing it with $ax^2 + bx + c = 0$, we have

$a = 1,$ $b = -4,$ $c = k$

For distinct real roots, we have

Discriminant > 0

⇨ $b^2 - 4ac > 0$

⇨ $(-4)^2 - 4 \times 1 \times k > 0$

⇨ $16 - 4k > 0$

⇨ $16 > 4k$

⇨ $4k < 16$

⇨ $k < \frac{16}{4}$

⇨ $k < 4$ (Ans)

CHAPTER – 7
COORDINATE GEOMETRY

21. Find the values of $\boldsymbol{a}$ when the distance between $\boldsymbol{P(a,-1)}$ ***and*** $\boldsymbol{Q(5,3)}$ ***is*** $\boldsymbol{5}$ ***units***.

Soln.: We have,

$PQ = 5$

⇨ $PQ^2 = 5^2$ [Squaring both sides]

⇨ $(5-a)^2 + (3+1)^2 = 5^2$

⇨ $(5-a)^2 + (4)^2 = 25$

⇨ $(5-a)^2 + 16 = 25$

⇨ $(5-a)^2 = 25 - 16$

⇨ $(5-a)^2 = 9$

⇨ $5 - a = \pm\sqrt{9}$

⇨ $5 - a = \pm 3$

⇨ $5 - a = 3 \quad or, \quad 5 - a = -3$

⇨ $a = 5 - 3 \quad or, \quad a = 5 + 3$

⇨ $a = 2 \quad or, \quad a = 8$

Hence, the values of a are 2 *and* 8. (Ans)

22. If the points $(2,1)$ *and* $(1,-2)$ are equidistant from the point (x,y), prove that $x + 3y = 0$.

Soln.: Since the point $P(x,y)$ is equidistant from the points $A(2,1)$ & $B(1,-2)$.

$\therefore AP = PB$

⇨ $AP^2 = PB^2$ [*Squaring both sides*]

⇨ $(x-2)^2 + (y-1)^2 = (x-1)^2 + (y+2)^2$

⇨ $x^2 - 4x + 4 + y^2 - 2y + 1 = x^2 - 2x + 1 + y^2 + 4y + 4$

⇨ $-4x - 2y = -2x + 4y$

⇨ $-4x + 2x = 4y + 2y$

⇨ $-2x = 6y$

⇨ $-x = 3y$

⇨ $0 = x + 3y$

⇨ $x + 3y = 0$

Hence, $x + 3y = 0$. (Proved)

23. Find the coordinates of the point, which divides the join of $A(-1,7)$ *and* $B(4,-3)$ in the ratio $2:3$.

Soln.: Let the coordinates of the point be $P(x,y)$

Here, $x_1 = -1$, $y_1 = 7$

$x_2 = 4$, $y_2 = -3$

$m = 2$, $n = 3$

Now, Using Section Formula, we have

$$(x, y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$
$$= \left(\frac{2\times4+3(-1)}{2+3}, \frac{2(-3)+3\times7}{2+3}\right)$$
$$= \left(\frac{8-3}{5}, \frac{-6+21}{5}\right)$$
$$= \left(\frac{5}{5}, \frac{15}{5}\right)$$
$$= (1, 3)$$

∴ The coordinates of the required point is $P(1, 3)$. (Ans)

24. Find the area of the triangle whose vertices are $(5, -7)$, $(-4, -5)$ *and* $(4, 5)$.

Soln.: Let the given vertices be $A(5, -7)$, $B(-4, -5)$ *and* $C(4, 5)$

Here, $x_1 = 5$, $y_1 = -7$

$x_2 = -4$, $y_2 = -5$

$x_3 = 4$, $y_3 = 5$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$
$$= \frac{1}{2}|5(-5-5) + (-4)\{5-(-7)\} + 4\{-7-(-5)\}|$$
$$= \frac{1}{2}|5(-10) + (-4)(5+7) + 4(-7+5)|$$
$$= \frac{1}{2}|-50 + (-4)(12) + 4(-2)|$$
$$= \frac{1}{2}|-50 - 48 - 8|$$
$$= \frac{1}{2}|-106|$$
$$= \frac{1}{2} \times 106$$
$$= 53 \text{ Sq. units.} \quad \text{(Ans)}$$

25. Find the third vertex of a triangle ABC, if two of its vertices are $B(-3, 1)$ and $C(0, -2)$ and its centroid is at the origin.

Soln.: Let the third vertex of a triangle be $A(x_3, y_3)$ and its centroid is $(0, 0)$ at the origin.

Here, $x_1 = -3$, $y_1 = 1$

$x_2 = 0$, $y_2 = -2$

$x = 0$, $y = 0$

Now, Using the centroid formula, we have

$$(x, y) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$
$$\Rightarrow (0, 0) = \left(\frac{-3+0+x_3}{3}, \frac{1+(-2)+y_3}{3}\right)$$
$$\Rightarrow (0, 0) = \left(\frac{-3+x_3}{3}, \frac{-1+y_3}{3}\right)$$

Then,

$$0 = \frac{-3+x_3}{3}$$

⇨ $-3 + x_3 = 0$

⇨ $x_3 = 3$

And,

$$0 = \frac{-1+y_3}{3}$$

⇨ $-1 + y_3 = 0$

⇨ $y_3 = 1.$

Hence, the required third vertex is $(3, 1)$. (Ans)

26. Find the ratio in which the point $P(m, 6)$ divides the line segment joining the points $A(-1, 3)$ *and* $B(2, 8)$. Also, find the value of m.

Soln.: Let $P(m, 6)$ divides the line segment joining $A(-1, 3)$ *and* $B(2, 8)$ in the ratio $k: 1$.

Here, $x_1 = -1$, $y_1 = 3$

$x_2 = 2$, $y_2 = 8$

$m = k$, $n = 1$

Now, Using Section Formula, we have coordinates of P

$$= \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

⇨ $(m, 6) = \left(\frac{k\times2+1(-1)}{k+1}, \frac{k\times8+1\times3}{k+1}\right)$

⇨ $(m, 6) = \left(\frac{2k-1}{k+1}, \frac{8k+3}{k+1}\right)$

Now, $6 = \frac{8k+3}{k+1}$

⇨ $6k + 6 = 8k + 3$

⇨ $6k - 8k = 3 - 6$

⇨ $-2k = -3$

⇨ $k = \frac{3}{2}$

⇨ $k: 1 = 3: 2$

And, $m = \frac{2k-1}{k+1}$

⇨ $m = \frac{2\times\frac{3}{2}-1}{\frac{3}{2}+1}$

⇨ $m = \frac{3-1}{\frac{5}{2}}$

⇨ $m = \frac{4}{5}$

Hence, the required ratio is $3: 2$ *and* $m = \frac{4}{5}$. (Ans)

27. Find the value of p for which the points $A(-1, 3)$, $B(2, p)$ *and* $C(5, -1)$ are collinear.

Soln.: Here, $x_1 = -1$, $y_1 = 3$

$x_2 = 2$, $y_2 = p$

$x_3 = 5$, $y_3 = -1$

Since, the given points are collinear

$\therefore$ ***Area of*** Δ ***ABC*** $= \mathbf{0}$

⇨ $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$

⇨ $\frac{1}{2}|(-1)\{p - (-1)\} + 2(-1 - 3) + 5(3 - p)| =$

⇨ $\frac{1}{2}|(-1)(p + 1) + 2(-4) + 15 - 5p| = 0$

⇨ $\frac{1}{2}|-p - 1 - 8 + 15 - 5p| = 0$

⇨ $\frac{1}{2}|-6p + 6| = 0$

⇨ $-6p + 6 = 0$

⇨ $-6p = -6$

⇨ $p = 1$ (Ans)

28. If the point $P(-1, 2)$ divides the line segment joining $A(2, 5)$ *and* B in the ratio $3:4$, find the coordinates of B.

Soln.: Let $P(-1, 2)$ divide the line segment AB in the ratio $3:4$.

By Section Formula, the coordinates of P are -

$$\left(\frac{3x+4(2)}{3+4}, \frac{3y+4(5)}{3+4}\right)$$

$$= \left(\frac{3x+8}{7}, \frac{3y+20}{7}\right)$$

But, P *is* $(-1, 2)$

$\therefore \frac{3x+8}{7} = -1$ *and* $\frac{3y+20}{7} = 2$

⇨ $3x + 8 = -7$ *and* $3y + 20 = 14$

⇨ $3x = -7 - 8$ *and* $3y = 14 - 20$

⇨ $x = -\frac{15}{3}$ *and* $y = -\frac{6}{3}$

⇨ $x = -5$ *and* $y = -2$

Hence, the coordinates of B *are* $(-5, -2)$. (Ans)

29. Prove that the points $(3, 0)$, $(6, 4)$ *and* $(-1, 3)$ are the vertices of a right-angled isosceles triangle.

Soln.: Let the given points be $A(3, 0)$, $B(6, 4)$ *and* $C(-1, 3)$.

Then, By Distance Formula, we have

$AB = \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ *units*

$BC = \sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2}$ *units*

$AC = \sqrt{(-1-3)^2 + (3-0)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$ *units*

Thus, $AB = AC$ [5 *units each*]

⇨ ΔABC is an isosceles triangle.

Also,

$BC^2 = 50 = 25 + 25 = AB^2 + AC^2$

⇨ $\angle A = 90°$ [$Pythagoras\ Theorem$]

⇨ $\Delta\, ABC$ is right angled at A.

Hence, the given points are the vertices of a right-angled isosceles triangle.

Proved.

30. Find a point on the $y - axis$ which is equidistant from the points $A(2, 3)$ and $B(-4, 1)$.

Soln.: We know that a point on the $y - axis$ is of the form $(0, y)$.

So, Let the point $P(0, y)$ be equidistant from the points $A(2, 3)$ and $B(-4, 1)$.

Then,

$PA = PB$

⇨ $PA^2 = PB^2$ [Squaring both sides]

⇨ $(2 - 0)^2 + (3 - y)^2 = (-4 - 0)^2 + (1 - y)^2$

⇨ $(2)^2 + (3)^2 - 2(3)y + y^2 = (-4)^2 + (1)^2 - 2(1)y + y^2$

⇨ $4 + 9 - 6y + y^2 = 16 + 1 - 2y + y^2$

⇨ $13 - 6y = 17 - 2y$

⇨ $-6y + 2y = 17 - 13$

⇨ $-4y = 4$

⇨ $y = \frac{4}{-4}$

⇨ $y = -1$

Hence, the required point on the $y - axis$ is $(0, -1)$. (Ans)

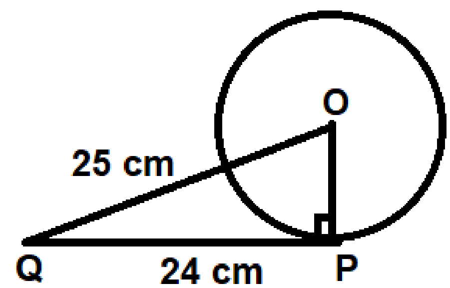
CIRCLES

31. If the tangent at a point P to a circle with centre O cuts a line through O at Q such that $PQ = 24cm\ and\ OQ = 25cm$. Find the radius of the circle.

Soln.: Tangent $PQ = 24\ cm\ and\ OQ = 25\ cm$

Radius OP is joined.

Since, the tangent at any point of a circle is perpendicular to the radius through the point of contact.



$\therefore OP \perp PQ \Rightarrow \angle OPQ = 90°$

Now, in rt. $\Delta\ OPQ$,

By Pythagoras Theorem, we have

$$OQ^2 = OP^2 + PQ^2$$

⇨ $(25)^2 = OP^2 + (24)^2$

⇨ $625 = OP^2 + 576$

⇨ $OP^2 = 625 - 576$

⇨ $OP^2 = 49$

⇨ $OP = \sqrt{49}$

⇨ $OP = 7\ cm$

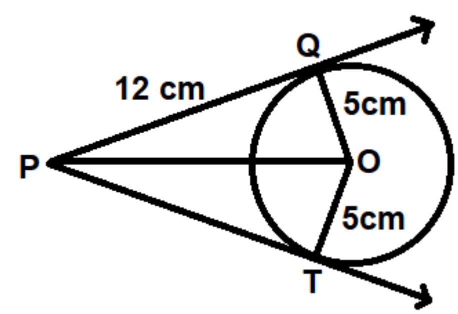
Hence, the radius of the circle is $7\ cm$. (Ans)

32. PQ and PT are tangents to a circle with centre O and radius $5\ cm$. If $PQ = 12\ cm$, then prove that the perimeter of the quadrilateral is $34\ cm$.

Soln.: Radius $= OQ = OT = 5\ cm$

Tangent, $PQ = 12\ cm$

Since, the lengths of two tangents drawn from an external point to a circle are equal.



$\therefore PT = PQ = 12\ cm$

Now,

Perimeter of quadrilateral $PQOT$

$= PQ + OQ + OT + TP$

$= 12 + 5 + 5 + 12$

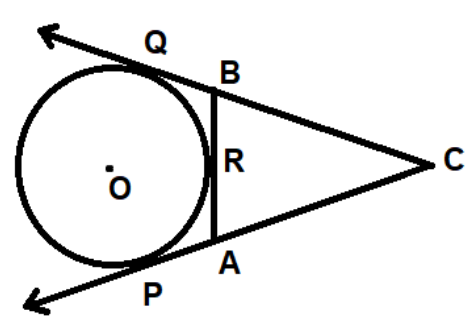
$= 34\ cm$. Hence, Proved.

33. In the figure, $CP\ and\ CQ$ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If $CP = 11\ cm\ and\ BC = 7\ cm$, then find the length of BR.

Soln.: Given: $CP = 11\ cm\ and\ BC = 7\ cm$.

Since, the lengths of two tangents drawn from an external point to a circle are equal.

$\therefore CP = CQ = 11\ cm$ -------------(i)



& $BQ = BR$ --------------(ii)

But, $CQ = BQ + BC$

⇨ $11 = BR + 7$ [from (i) & (ii)]

⇨ $11 - 7 = BR$

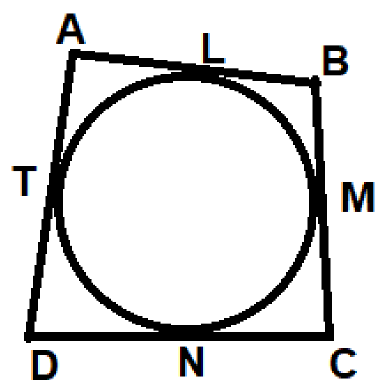
⇨ $4 = BR$

⇨ $BR = 4\ cm$

Hence, the length of BR is $4\ cm$. (Ans)

34. In the figure, quadrilateral $ABCD$ is circumscribed. Find the perimeter of a quadrilateral $ABCD$, if $AL = 6\ cm,\ BL = 5\ cm,\ CM = 3\ cm\ and\ DN = 4\ cm$.

<u>Soln.:</u>



Since, the lengths of two tangents drawn from an external point to a circle are equal.

∴ $AL = AT = 6\ cm\ (Tangents\ from\ A)$ -----(i)

$BL = BM = 5\ cm\ (Tangents\ from\ B)$ -----(ii)

$CM = CN = 3\ cm\ (Tangents\ from\ C)$ -----(iii)

$DN = DT = 4\ cm\ (Tangents\ from\ D)$ -----(iv)

Now, Perimeter of a quadrilateral $ABCD$

$= AB + BC + CD + DA$

$= (AL + BL) + (BM + CM) + (DN + CN) + (AT + L$

[$from\ (i), (ii), (iii) \& (iv)$]

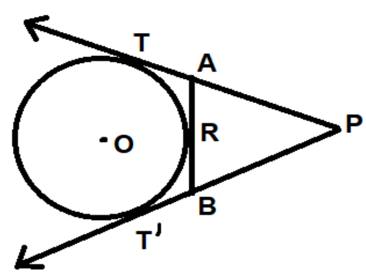
$= (6 + 5) + (5 + 3) + (4 + 3) + (6 + 4)$

$= 11 + 8 + 7 + 10$

$= 36\ cm$. (Ans)

35. In the figure, $PT\ and\ PT'$ are tangents from P to the circle with centre O. R is a point on the circle. Prove that $PA + AR = PB + BR$.

<u>Soln.:</u>



Since, the lengths of two tangents drawn from an external point to a circle are equal.

Then, $PT = PT'\ (Tangents\ from\ P)$-----(i)

$AT = AR\ (Tangents\ from\ A)$ -----(ii)

& $BT' = BR\ (Tangents\ from\ B)$ -----(iii)

From equation (i), we have

$PT = PT'$

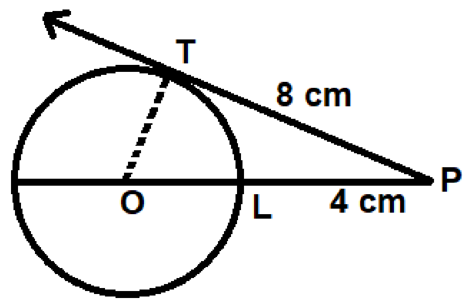
⇨ $PA + AT = PB + BT'$

⇨ $PA + AR = PB + BR$ [Using (ii) & (iii)]. Hence, Proved

36. In the figure, $\boldsymbol{O}$ is the centre of the circle, $\boldsymbol{PT}$ is the tangent and $\boldsymbol{PLM}$ is the secant passing through the centre $\boldsymbol{O}$. If $\boldsymbol{PT = 8\ cm\ and\ PL = 4\ cm}$, then find the radius of the circle.

Soln.: Given: Tangent $\boldsymbol{PT = 8\ cm}$

$\boldsymbol{PL = 4\ cm\ and\ OT\ is\ joined.}$



Let, Radius $= \boldsymbol{OT = OL = r}$

Since, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \boldsymbol{OT \perp PT \Rightarrow \angle OTP = 90°}$

Now, in rt. $\Delta\ \boldsymbol{OTP}$,

By Pythagoras Theorem, we have

$$\boldsymbol{OP^2 = OT^2 + PT^2}$$

⇨ $\boldsymbol{(OL + PL)^2 = r^2 + (8)^2}$

⇨ $\boldsymbol{(r + 4)^2 = r^2 + 64}$

⇨ $\boldsymbol{r^2 + 8r + 16 = r^2 + 64}$

⇨ $\boldsymbol{8r + 16 = 64}$

⇨ $\boldsymbol{8r = 64 - 16}$

⇨ $\boldsymbol{8r = 48}$

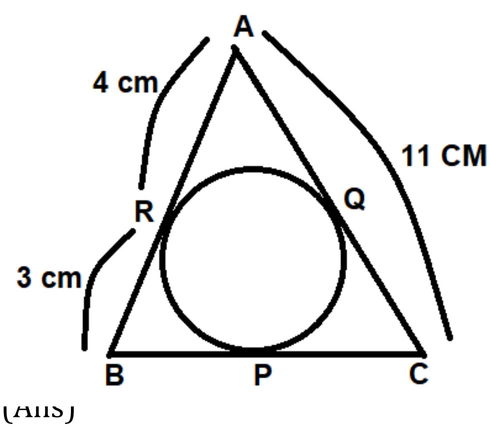
⇨ $\boldsymbol{r = \frac{48}{8}}$

⇨ $\boldsymbol{r = 6\ cm}$

Hence, the radius of the circle is $\boldsymbol{6\ cm}$. (Ans)

37. In the figure, $\Delta\ \boldsymbol{ABC}$ is circumscribing a circle. Find the length of $\boldsymbol{BC}$.

Soln.:



Since, the lengths of two tangents drawn from an external point to a circle are equal.

$\therefore\ \boldsymbol{AR = AQ = 4\ cm}$

$\boldsymbol{BR = BP = 3\ cm}$

& $\boldsymbol{PC = QC}$

⇨ $\boldsymbol{PC = AC - AQ}$

⇨ $\boldsymbol{PC = 11 - 4}$

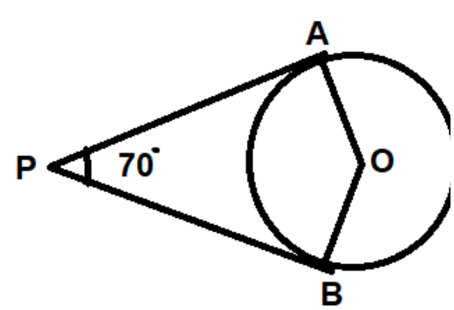
⇨ $\boldsymbol{PC = 7\ cm.}$

Hence, the length of $\boldsymbol{BC = (BP + PC) = 3 + 7 = 10\ cm.}$ (Ans)

38. The two tangents drawn from an external point $\boldsymbol{P}$ to a circle with centre $\boldsymbol{O}$ are $\boldsymbol{PA}$ and $\boldsymbol{PB}$. If $\boldsymbol{\angle APB = 70°}$, what is the value of $\boldsymbol{\angle AOB}$?

Soln.:

Since, the tangent at any point of a circle is perpendicular to the radius through the point of contact.



$\therefore\ \boldsymbol{AO \perp AP \Rightarrow \angle PAO = 90°}$

$\boldsymbol{OB \perp PB \Rightarrow \angle PBO = 90°}$

Now, in quadrilateral $\boldsymbol{PAOB}$, we have

$$\boldsymbol{\angle PAO + \angle AOB + \angle PBO + \angle APB = 360°}$$

⇨ $\boldsymbol{90° + \angle AOB + 90° + 70° = 360°}$

⇨ $\boldsymbol{\angle AOB + 250° = 360°}$

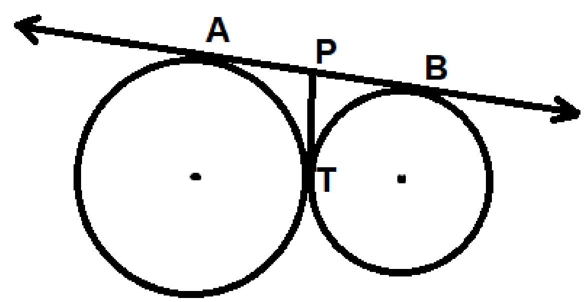
⇨ $\boldsymbol{\angle AOB = 360° - 250°}$

⇨ $\boldsymbol{\angle AOB = 110°}$ (Ans)

39. In the figure, $\boldsymbol{AB}$ is a common tangent to the given circles, which touch externally at $\boldsymbol{P}$. If $\boldsymbol{AP = 3.2\ cm}$, find the length of $\boldsymbol{AB}$.

Soln.: Given: $\boldsymbol{AP = 3.2\ cm}$

Since, the lengths of two tangents drawn from an external point to a circle are equal.



$\therefore\ \boldsymbol{PT = PA = 3.2\ cm}$

& $\boldsymbol{PT = PB}$

$\therefore\ \boldsymbol{PA = PB = 3.2\ cm}$

Now,

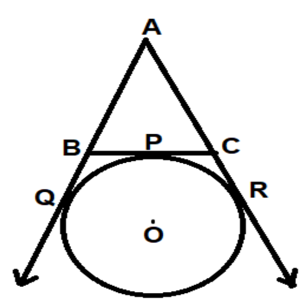
$$\boldsymbol{AB = PA + PB = 3.2 + 3.2 = 6.4\ cm}$$

Hence, the length of $\boldsymbol{AB}$ is $\boldsymbol{6.4\ cm}$. (Ans)

40. A circle touches the side $\boldsymbol{BC}$ of a $\Delta\ \boldsymbol{ABC}$ at a point $\boldsymbol{P}$ and touches $\boldsymbol{AB}$ and $\boldsymbol{AC}$ when produced at $\boldsymbol{Q}$ and $\boldsymbol{R}$ respectively. Show that $\boldsymbol{AQ = \frac{1}{2}(Perimeter\ of\ \Delta ABC)}$.

Soln.:

Since, the lengths of two tangents drawn from an external point to a circle are equal.



$\therefore\ \boldsymbol{BP = BQ}$ --------(i)

$\boldsymbol{CP = CR}$ --------(ii)

& $\boldsymbol{AQ = AR}$ --------(iii)

Now,

$$\begin{aligned}\text{Perimeter of } \Delta\ \boldsymbol{ABC} &= \boldsymbol{AB + BC + CA}\\ &= \boldsymbol{AB + (BP + PC) + CA}\\ &= \boldsymbol{(AB + BP) + (PC + CA)}\\ &= \boldsymbol{(AB + BQ) + (CR + CA)[from\ (i)\&(ii)]}\\ &= \boldsymbol{AQ + AR}\\ &= \boldsymbol{AQ + AQ\ [from\ (iii)]}\\ &= \boldsymbol{2AQ}\end{aligned}$$

Hence, $\boldsymbol{AQ = \frac{1}{2}(Perimeter\ of\ \Delta\ ABC)}$. (Shown)

CHAPTER – 12

AREAS RELATED TO CIRCLES

41. The difference between the circumference and the radius of a circle is $\mathbf{37\ cm}$. Find the area of the circle. $\left(\boldsymbol{Use\ \pi = \frac{22}{7}}\right)$

Soln.: Let the radius of the circle be '$\boldsymbol{r}$' cm.

Now, According to the question, we have

Circumference – Radius $= \mathbf{37\ cm}$

⇨ $\boldsymbol{2\pi r - r = 37}$

⇨ $\boldsymbol{r(2\pi - 1) = 37}$

⇨ $\boldsymbol{r\left(2 \times \frac{22}{7} - 1\right) = 37}$

⇨ $\boldsymbol{r\left(\frac{44-7}{7}\right) = 37}$

⇨ $\boldsymbol{r \times \frac{37}{7} = 37}$

⇨ $\boldsymbol{r = 37 \times \frac{7}{37}}$

⇨ $\boldsymbol{r = 7\ cm}$

Hence, Area of the circle $= \boldsymbol{\pi r^2}$

$= \boldsymbol{\frac{22}{7} \times 7^2}$

$= \boldsymbol{\frac{22}{7} \times 49}$

$= \mathbf{22 \times 7}$

$= \mathbf{154\ cm^2}$ (Ans)

42. Find the angle subtended at the centre of a circle of radius $\mathbf{5\ cm}$ by an arc of length $\boldsymbol{\frac{5\pi}{3}\ cm}$.

Soln.: Given: Radius, $\boldsymbol{r = 5\ cm}$

Length of arc, $\boldsymbol{l = \frac{5\pi}{3} cm}$

Now, Length of an arc $= \boldsymbol{\frac{5\pi}{3}}$

⇨ $\boldsymbol{\frac{\pi r\theta}{180°} = \frac{5\pi}{3}}$

⇨ $\boldsymbol{\theta = \frac{5\pi \times 180°}{3 \times \pi \times 5}}$

⇨ $\boldsymbol{\theta = 60°}$

Hence, the required angle is $\mathbf{60°}$. (Ans)

43. A race track is in the form of a ring whose inner circumference is $\boldsymbol{352\ m}$ and the outer circumference is $\boldsymbol{396\ m}$. Find the width of the track.

Soln.: Let the inner radius be $'\boldsymbol{r}'$

& the outer radius be $'\boldsymbol{R}'$

Given:

Inner Circumference $= \boldsymbol{352\ m}$

⇨ $\boldsymbol{2\pi r = 352}$

⇨ $\boldsymbol{2 \times \frac{22}{7} \times r = 352}$

⇨ $\boldsymbol{r = \frac{352\times7}{2\times22}}$

⇨ $\boldsymbol{r = 56\ m}$

Also,

Outer Circumference $= \boldsymbol{396\ m}$

⇨ $\boldsymbol{2\pi R = 396}$

⇨ $\boldsymbol{2 \times \frac{22}{7} \times R = 396}$

⇨ $\boldsymbol{R = \frac{396\times7}{2\times22}}$

⇨ $\boldsymbol{R = 63\ m}$

∴ The width of the track $= \boldsymbol{R - r}$

$= \boldsymbol{63 - 56}$

$= \boldsymbol{7\ m}$ (Ans)

44. Find the area of the minor sector of a circle of radius $\boldsymbol{4\ cm}$ and of angle $\boldsymbol{30°}$. $(\boldsymbol{Use\ \pi = 3.14})$

Soln.: Given: Radius of the sector, $\boldsymbol{r = 4\ cm}$

& Sector angle , $\boldsymbol{\theta = 30°}$

Now,

Area of the minor sector $= \boldsymbol{\frac{\pi r^2 \theta}{360°}}$

$= \boldsymbol{\frac{3.14\times(4)^2\times30}{360}}$

$= \boldsymbol{\frac{3.14\times16\times30}{360}}$

$= \boldsymbol{\frac{3.14\times16}{12}}$

$= \boldsymbol{\frac{3.14\times4}{3}}$

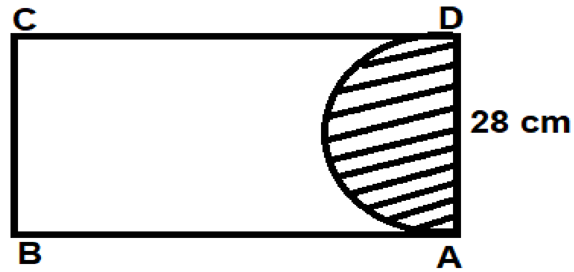
$= \boldsymbol{\frac{12.56}{3}}$

$= \boldsymbol{4.186\ cm^2}$

$= 4.\boldsymbol{19\ cm^2}$ (Ans)

45. A sheet of paper is in the form of a rectangle $\boldsymbol{ABCD}$ in which $\boldsymbol{AB = 40\ cm}$ and $\boldsymbol{AD = 28\ cm}$ as shown in the adjoining figure. A semi-circular portion with $\boldsymbol{AD}$ as a diameter is cut off. Find the area of the remaining paper. $\left(\boldsymbol{Use\ \pi = \frac{22}{7}}\right)$

Soln.:



Radius, $r = \frac{AD}{2} = \frac{28}{2} = 14\ cm$

Now,

Area of the remaining paper

$= \boldsymbol{Area\ of\ the\ rectangle - Area\ of\ the\ semi\ circle}$

$= \boldsymbol{AB \times AD - \frac{1}{2}\pi r^2}$

$= \boldsymbol{40 \times 28 - \frac{1}{2} \times \frac{22}{7} \times (14)^2}$

$= \boldsymbol{1120 - \frac{1}{2} \times \frac{22}{7} \times 14 \times 14}$

$= \boldsymbol{1120 - 11 \times 2 \times 14}$

$= \boldsymbol{1120 - 308}$

$= \boldsymbol{812\ cm^2}$ (Ans)

46. What is the perimeter of a sector of angle $\boldsymbol{45°}$ of a circle with radius $\boldsymbol{7\ cm}$. $\left(\boldsymbol{Use\ \pi = \frac{22}{7}}\right)$

Soln.: Given: Radius $(\boldsymbol{r}) = \boldsymbol{7\ cm}$

Central angle $(\boldsymbol{\theta}) = \boldsymbol{45°}$

Now,

Perimeter of the sector $= \boldsymbol{2r + \frac{\pi r \theta}{180°}}$

$= \boldsymbol{2 \times 7 + \frac{22 \times 7 \times 45°}{7 \times 180°}}$

$= \boldsymbol{14 + \frac{11}{2}}$

$= \boldsymbol{14 + 5.5}$

$= \boldsymbol{19.5\ cm}$ (Ans)

47. A circular park, $\boldsymbol{42\ m}$ in diameter, has a path $\boldsymbol{3.5\ m}$ wide running around it on the outside. Find the cost of gravelling the path at Rs $\boldsymbol{20}$ per sq.m. $\left(\boldsymbol{Use\ \pi = \frac{22}{7}}\right)$

Soln.: Inner diameter $(\boldsymbol{d}) = \boldsymbol{42\ m}$

$\therefore$ Inner Radius $(\boldsymbol{r}) = \boldsymbol{\frac{d}{2} = \frac{42}{2} = 21\ m}$

So, Radius of outer circle $(R) = 21\ m + 3.5\ m = 24.5\ m$

Now,

Area of the path $= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$

$= \pi(R + r)(R - r)$

$= \frac{22}{7}(24.5 + 21)(24.5 - 21)$

$= \frac{22}{7} \times 45.5 \times 3.5$

$= \frac{22 \times 455 \times 35}{7 \times 10 \times 10}$

$= 500.5\ m^2$

∴ The cost of graveling the path $= Rs\ 20 \times 500.5 = Rs\ 10010$. (Ans)

48. A chord of a circle of radius $14\ cm$ subtends an angle $60°$ at the centre. Find the area of the major sector. $\left(Use\ \pi = \frac{22}{7}\right)$

Soln.: Given: Radius $(r) = 14\ cm$

Sector angle $(\theta) = 60°$

Now,

Area of the major sector $= \pi r^2 - \frac{\pi r^2 \theta}{360°}$

$= \pi r^2\left(1 - \frac{\theta}{360°}\right)$

$= \frac{22}{7} \times (14)^2 \times \left(1 - \frac{60°}{360°}\right)$

$= \frac{22}{7} \times 14 \times 14 \times \left(1 - \frac{1}{6}\right)$

$= 22 \times 2 \times 14 \times \left(\frac{5}{6}\right)$

$= \frac{3080}{6}$

$= 513.33\ cm^2$ (Ans)

49. A steel wire when bent in the form of a square encloses an area of $121\ cm^2$. If the same wire is bent in the form of a circle, find the area of the circle. $\left(Use\ \pi = \frac{22}{7}\right)$

Soln.: Let the radius of a circle be $'r'$.

Given:

Area of the square $= 121\ cm^2$

⇨ $(Side)^2 = 121$

⇨ $Side = \sqrt{121}$

⇨ $Side = 11\ cm$

∴ Perimeter of the square $= 4 \times Side = 4 \times 11 = 44\ cm$

Since, the same wire is bent in the form of a circle.

∴ Circumference of a circle = Perimeter of a square

⇨ $2\pi r = 44$

⇨ $2 \times \frac{22}{7} \times r = 44$

⇨ $r = \frac{44 \times 7}{2 \times 22}$

⇨ $r = 7\ cm$

Hence, Area of the circle $= \pi r^2$

$= \frac{22}{7} \times (7)^2$

$= \frac{22}{7} \times 7 \times 7$

$= 22 \times 7$

$= 154\ cm^2$ (Ans)

50. A pendulum moving through an angle of $30°$ and describing an arc $4.4\ cm$ in length. Find the length of the pendulum. $\left(Use\ \pi = \frac{22}{7}\right)$

Soln.: Given: Sector angle $(\theta) = 30°$

Length of an arc $(l) = 4.4\ cm$

Let the length of the pendulum be $'r'$.

Now,

Length of an arc $= 4.4\ cm$

⇨ $\frac{\pi r \theta}{180°} = 4.4$

⇨ $\frac{22 \times r \times 30°}{7 \times 180°} = \frac{44}{10}$

⇨ $\frac{22 \times r}{7 \times 6} = \frac{44}{10}$

⇨ $r = \frac{44 \times 7 \times 6}{10 \times 22}$

⇨ $r = 8.4\ cm$

Hence, the length of the pendulum is $8.4\ cm$ (Ans)

CHAPTER – 14
STATISTICS

51. Find the mean of the following data:

Class Interval	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**
Frequency	**12**	**16**	**6**	**7**	**9**

Soln.:

Class Interval	Frequency (f_i)	Class Mark (x_i)	$f_i x_i$
0 – 10	**12**	**5**	**60**
10 – 20	**16**	**15**	**240**
20 – 30	**6**	**25**	**150**
30 – 40	**7**	**35**	**245**
40 – 50	**9**	**45**	**405**
	$\sum f_i = 50$		$\sum f_i x_i = 1100$

By Using Direct Method, we have

$$Mean, \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{1100}{50}$$

$$= 22. \quad \text{(Ans)}$$

52. Find the mode of the following distribution:

Class Interval	**0 – 20**	**20 – 40**	**40 – 60**	**60 – 80**
Frequency	**15**	**6**	**18**	**10**

Soln.: Here, the class **40 – 60** has the maximum frequency **18.**

So, the modal class is **40 – 60.**

Therefore, Lower limit (l) of the modal class = **40**

Class size (h) = **20**

Frequency (f_1) of the modal class = **18**

Frequency (f_0) of the class preceding the modal class = **6**

Frequency (f_2) of the class succeeding the modal class = **10**

Now, Using the formula, we have

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{18-6}{2\times18-6-10}\right) \times 20$$

$$= 40 + \left(\frac{12}{36-16}\right) \times 20$$

$$= 40 + \frac{12}{20} \times 20$$

$$= 40 + 12$$

$$= 52 \quad \text{(Ans)}$$

53. Find the median of the following frequency distribution:

Class Interval	$\mathbf{0-100}$	$\mathbf{100-200}$	$\mathbf{200-300}$	$\mathbf{300-400}$	$\mathbf{400-500}$
Frequency	$\mathbf{40}$	$\mathbf{32}$	$\mathbf{48}$	$\mathbf{22}$	$\mathbf{8}$

Soln.:

Class Interval	Frequency	Cumulative Frequency
$\mathbf{0-100}$	$\mathbf{40}$	$\mathbf{40}$
$\mathbf{100-200}$	$\mathbf{32}$	$\mathbf{40+32=72}$
$\mathbf{200-300}$	$\mathbf{48}$	$\mathbf{72+48=120}$
$\mathbf{300-400}$	$\mathbf{22}$	$\mathbf{120+22=142}$
$\mathbf{400-500}$	$\mathbf{8}$	$\mathbf{142+8=150}$
Total	$\boldsymbol{N=150}$	

Here, $\boldsymbol{N=150}$

Now, $\frac{\boldsymbol{N}}{\mathbf{2}}=\frac{\mathbf{150}}{\mathbf{2}}=\mathbf{75}$

The cumulative frequency just greater than $\mathbf{75}$ is $\mathbf{120}$ which corresponds to the class $\mathbf{200-300}$.

$\therefore$ The median class is $\mathbf{200-300}$

Here, $\boldsymbol{l}$ = Lower limit of the median class = $\mathbf{200}$

$\boldsymbol{N}$ = number of observation= $\mathbf{150}$

$\boldsymbol{C}$ = cumulative frequency of the class preceding the median class = $\mathbf{72}$

$\boldsymbol{f}$ = frequency of the median class = $\mathbf{48}$

$\boldsymbol{h}$ = class size = $\mathbf{100}$

Now, Using the formula, we have

$$\text{Median} = \boldsymbol{l}+\left(\frac{\frac{\boldsymbol{N}}{\mathbf{2}}-\boldsymbol{C}}{\boldsymbol{f}}\right)\times \boldsymbol{h}$$

$$=\mathbf{200}+\left(\frac{\mathbf{75-72}}{\mathbf{48}}\right)\times\mathbf{100}$$

$$=\mathbf{200}+\left(\frac{\mathbf{3}}{\mathbf{48}}\right)\times\mathbf{100}$$

$$=\mathbf{200}+\frac{\mathbf{1}}{\mathbf{16}}\times\mathbf{100}$$

$$=\mathbf{200}+\frac{\mathbf{25}}{\mathbf{4}}$$

$$=\mathbf{200+6.25}$$

$$=\mathbf{206.25}\quad \text{(Ans)}$$

54. Find the median, if

$\boldsymbol{l}$ = Lower limit of the median class = $\mathbf{15}$

$\boldsymbol{N}$ = Total observation= $\mathbf{49}$

$\boldsymbol{C}$ = cumulative frequency of the class preceding the median class = $\mathbf{11}$

$\boldsymbol{f}$ = frequency of the median class = $\mathbf{15}$

$\boldsymbol{h}$ = size of the class interval = $\mathbf{5}$

Soln.: Using the formula, we have

$$\boldsymbol{Median} = \boldsymbol{l} + \left(\frac{\frac{N}{2}-C}{f}\right) \times \boldsymbol{h}$$

$$= \mathbf{15} + \left(\frac{\frac{49}{2}-11}{15}\right) \times \mathbf{5}$$

$$= \mathbf{15} + \left(\frac{24.5-11}{15}\right) \times \mathbf{5}$$

$$= \mathbf{15} + \left(\frac{13.5}{15}\right) \times \mathbf{5}$$

$$= \mathbf{15} + \frac{13.5}{3}$$

$$= \mathbf{15} + \mathbf{4.5}$$

$$= \mathbf{19.5} \quad \text{(Ans)}$$

55. Find the mode, if

$\boldsymbol{l}$ = Lower limit of the modal class = $\mathbf{15}$

$\boldsymbol{h}$ = size of the class interval = $\mathbf{5}$

$\boldsymbol{f_1}$ = Frequency of the modal class = $\mathbf{24}$

$\boldsymbol{f_0}$ = Frequency of the class preceding the modal class = $\mathbf{18}$

$\boldsymbol{f_2}$ = Frequency of the class succeeding the modal class = $\mathbf{17}$

Soln.:Using the formula, we have

$$\boldsymbol{Mode} = \boldsymbol{l} + \left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right) \times \boldsymbol{h}$$

$$= \mathbf{15} + \left(\frac{24-18}{2\times24-18-17}\right) \times \mathbf{5}$$

$$= \mathbf{15} + \left(\frac{6}{48-35}\right) \times \mathbf{5}$$

$$= \mathbf{15} + \left(\frac{6}{13}\right) \times \mathbf{5}$$

$$= \mathbf{15} + \frac{30}{13}$$

$$= \mathbf{15} + \mathbf{2.3}$$

$$= \mathbf{17.3} \quad \text{(Ans)}$$

56. What is the difference of median and mean, if the difference of mode and median is 42 ?

Soln.: Given:

$$\boldsymbol{Mode} - \boldsymbol{Median} = \mathbf{42}$$

⇨ $\boldsymbol{Mode} = \mathbf{42} + \boldsymbol{Median}$ ------------------------(i)

Now, Relation among mean, median and mode is –

$\boldsymbol{Mode} = \mathbf{3}\boldsymbol{Median} - \mathbf{2}\boldsymbol{Mean}$ ----------------------(ii)

From equation (i) and (ii), we get

$$\mathbf{3}\ \boldsymbol{Median} - \mathbf{2}\ \boldsymbol{Mean} = \mathbf{42} + \boldsymbol{Median}$$

⇨ $\mathbf{3}\ \boldsymbol{Median} - \boldsymbol{Median} - \mathbf{2}\ \boldsymbol{Mean} = \mathbf{42}$

⇨ $\mathbf{2}\ \boldsymbol{Median} - \mathbf{2}\ \boldsymbol{Mean} = \mathbf{42}$

⇨ $\mathbf{2}(\boldsymbol{Median} - \boldsymbol{Mean}) = \mathbf{42}$

⇨ $\boldsymbol{Median} - \boldsymbol{Mean} = \frac{42}{2}$

⇨ $\boldsymbol{Median - Mean = 21}$. (Ans)

57. For the following distribution:

Marks	Number of students
$\boldsymbol{Below\ 10}$	$\mathbf{5}$
$\boldsymbol{Below\ 20}$	$\mathbf{7}$
$\boldsymbol{Below\ 30}$	$\mathbf{8}$
$\boldsymbol{Below\ 40}$	$\mathbf{12}$
$\boldsymbol{Below\ 50}$	$\mathbf{28}$
$\boldsymbol{Below\ 60}$	$\mathbf{30}$

Find the modal class.

Soln.:

Marks	Number of students	Cumulative Frequency $(\boldsymbol{cf})$
$\boldsymbol{Below\ 10}$	$\mathbf{5}$	$\mathbf{5}$
$\mathbf{10-20}$	$\mathbf{(7-5)=2}$	$\mathbf{7}$
$\mathbf{20-30}$	$\mathbf{(8-7)=1}$	$\mathbf{8}$
$\mathbf{30-40}$	$\mathbf{(12-8)=4}$	$\mathbf{12}$
$\mathbf{40-50}$	$\mathbf{(28-12)=16}$	$\mathbf{28}$
$\mathbf{50-60}$	$\mathbf{(30-28)=2}$	$\mathbf{30}$

So, we see that the highest frequency is $\mathbf{16}$, which lies in the class interval $\mathbf{40-50}$.

∴ The modal class is $\mathbf{40-50}$. (Ans)

58. The following frequency distribution gives the weights of 30 students of a class:

Weight (in Kg)	Number of students
$\mathbf{40-45}$	$\mathbf{2}$
$\mathbf{45-50}$	$\mathbf{3}$
$\mathbf{50-55}$	$\mathbf{8}$
$\mathbf{55-60}$	$\mathbf{6}$
$\mathbf{60-65}$	$\mathbf{6}$
$\mathbf{65-70}$	$\mathbf{3}$
$\mathbf{70-75}$	$\mathbf{2}$

Based on the above information, answer the following questions:

(i) Find the median class of the data.

(ii) Find the class mark of the median class.

Soln.:

Weight (in Kg)	Number of students $(\boldsymbol{f_i})$	Cumulative Frequency $(\boldsymbol{cf})$

$40-45$ $45-50$ $50-55$ $55-60$ $60-65$ $65-70$ $70-75$	2 3 8 6 6 3 2	2 $2+3=5$ $5+8=13$ $13+6=19$ $19+6=25$ $25+3=28$ $28+2=30$
Total	$N=30$	

Here, $N = 30$

⇨ $\frac{N}{2} = \frac{30}{2} = 15$ which is just greater than the cumulative frequency 13 and thus lies in the class $55 - 60$.

(i) So, the median class of the given data is $55 - 60$.

(ii) The class mark of the median class $55 - 60$ is

$$= \frac{Lower\ class\ limit + Upper\ class\ limit}{2}$$

$$= \frac{55 + 60}{2}$$

$$= \frac{115}{2}$$

$= 57.5$ (Ans)

59. For the following distribution:

Class	$0-10$	$10-20$	$20-30$	$30-40$	$40-50$	$50-60$
Frequency	3	6	8	15	10	8

Find the sum of lower limit of the median class and the lower limit of the modal class.

Soln.:

Class Interval	Frequency	Cumulative Frequency (cf)
$0-10$	3	3
$10-20$	6	$3+6=9$
$20-30$	8	$9+8=17$
$30-40$	15	$17+15=32$
$40-50$	10	$32+10=42$
$50-60$	8	$42+8=50$

Here, $N = 50$

⇨ $\frac{N}{2} = \frac{50}{2} = 25$,

which is just greater than the cumulative frequency 17 and thus lies in the class interval $30 - 40$.

So, Median class is $30 - 40$

∴ Lower limit of the median class is 30

Also, the highest frequency is 15, which lies in the class interval $30 - 40$.

So, Modal Class is $30 - 40$

∴ Lower limit of the modal class is $\mathbf{30}$

Hence, Required sum $= \mathbf{30 + 30 = 60}$. (Ans)

60. Find the mean of the following data:

x_i	13	15	17	19	21	23
f_i	8	2	3	4	5	6

Soln.:

x_i	f_i	$f_i x_i$
13	8	104
15	2	30
17	3	51
19	4	76
21	5	105
23	6	138
	$\sum f_i = 28$	$\sum f_i x_i = 504$

Now, Using the formula, we have

Mean, $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{504}{28} = \mathbf{18}$. (Ans)

Section-D

Long Answer Questions (5 Marks)

Arithmetic Progression

1. Which term of the arithmetic progression 3, 10, 17,…….. will be 84 more than the 13^{th} term?

   Solution: Given AP is 3, 10, 13….

   First term $a_1 = 3$

   Common difference, d = 10-3

   = 7

   Let the n^{th} term be 84 more than 13^{th} term

   ∴ $a_n - a_{13} = 84$

   ⇨ $a_1 + (n-1)d - \{a_1 + 12d\} = 84$

   ⇨ $a_1 + (n-1)d - a_1 - 12d = 84$

   ⇨ $(n-1) \times 7 - 12 \times 7 = 84$

   ⇨ $7n - 7 - 84 = 84$

   ⇨ $7n - 91 = 84$

   ⇨ $7n = 84 + 91$

   ⇨ $7n = 175$

   ⇨ $n = \frac{175}{7}$

   ⇨ $n = 25$

   Therefore, the 25^{th} is the required term.

2. Find the sum of the two -digit odd number.

   Solution: The first two-digit odd number is 11

   And the last two-digit odd number is 99.

   ∴ The arithmetic series is 11 + 13 + 15 + …….+99

   Now: First term, $a_1 = 11$, second term $a_2 = 13$, $a_3 = 15$, … , $a_n = 99$

   Common difference, $d = a_2 - a_1 = 13 - 11 = 2$

   Let n be the numbers of the two-digit odd numbers

   I, e, $a_n = 99$

   ⇨ $a_1 + (n-1)d = 99$

   ⇨ $11 + (n-1) \times 2 = 99$

   ⇨ $(n-1) \times 2 = 99 - 11$

   ⇨ $(n-1) \times 2 = 88$

   ⇨ $(n-1) = \frac{88}{2}$

   ⇨ $(n-1) = 44$

   ⇨ $n = 44 + 1$

   ⇨ $n = 45$

   Using the formula,

$$S_n = \frac{n}{2}(a_1 + a_n)$$

⇨ $S_{45}=\frac{45}{2}(11 + 99)$

⇨ $S_{45} = \frac{45}{2} \times 110$

⇨ $S_{45}=45x55$

⇨ $S_{45}=2475.$

∴ the sum of the two digit odd numbers is 2475.

3. If the 10th of an A.P is 52 and the 17th term is 20 more than the 13th term, find the A.P.
   Solution: let a_1 be the first term and 'd' the common difference.
   Given: $a_{10}=52$
   ⇨ $a_1+9d=52$ (i)
   And, $a_{17} - a_{13} =20$
   ⇨ $(a_1+16d)-(a_1+12d)=20$
   ⇨ $a_1+16d-a_1-12d =20$
   ⇨ $4d =20$
   ⇨ $d = \frac{20}{4}$
   ⇨ $d =5$
   putting d=5 in (i), we get,
   ⇨ $a_1+9x\ 5= 52$
   ⇨ $a_1+45 = 52$
   ⇨ $a_1 =52-45$
   ⇨ $a_1 =7$
   Hence the A.P is 7,7+5,7+2x5,7+3x5,…
   i.e 7,12,17,22,…
   Thus, the A.P is 7,12,17,22,…
4. The sum of the 5th term and the 9th term of an AP is 30. If its 25th term is three times its 8th term, find the AP.
   Solution: Let a_1 be the first term and d be the common difference
   Given: $a_5 + a_9 = 30$
   ⇨ $a_1 + 4d + a_1 + 8d = 30$
   ⇨ $2a_1 + 12d = 30$
   ⇨ $2(a_1 + 6d) = 30$
   ⇨ $a_1 + 6d = 15$ (i)

   And, $a_{25} = 3\times a_8$

   ⇨ $a_1 + 24d = 3(a_1 + 7d)$
   ⇨ $a_1 + 24d = 3a_1 + 21d$
   ⇨ $a_1 -3a_1+ 24d – 21d = 0$
   ⇨ $-2a_1 + 3d =0$ (ii)
   Multiplying equation (i) by 2 and equation (ii) by 1 we get
   $$2a_1 + 12d = 30$$

$$-2a_1 + 3d = 0$$

By adding $\quad 15d = 30$

$$d = \frac{30}{15} = 2$$

Putting d = 2 in equation (i) we get

$$a_1 + 6\times2 = 15$$

⇨ $a_1 + 12 = 15$

⇨ $a_1 = 15-12$

⇨ $a_1 = 3$

Thus the AP is 3, 3+2, 3+2×2, 3 + 2×3,…..

I.e., 3, 5, 7, 9,……

Thus the AP is 3, 5, 7, 9,……

5. The 2nd, 31st and the last term of an AP are $\mathbf{7\frac{3}{4}}$, $\frac{1}{2}$ and $\mathbf{-6\frac{1}{2}}$ respectively. Find the first term and the number of terms.

Solution: Let a_1 be the first term and d be the common difference and n be the number of terms of the AP.

Given: $\quad a_2 = 7\frac{3}{4}$, $a_{31} = \frac{1}{2}$ and $a_n = -6\frac{1}{2}$ or $-\frac{13}{2}$

⇨ $a_2 = \frac{34}{4}$

⇨ $a_1 + d = \frac{31}{4}$

⇨ $4a_1 + 4d = 31 \quad$ (i)

$a_{31} = \frac{1}{2}$

⇨ $a_1 + 30d = \frac{1}{2}$

⇨ $2a_1 + 60d = 1 \quad$ (i)

Multiplying equation (i) by 2 and equation (ii) by -4 we get.

$$8a_1 + 8d = 62$$

$$-8a_1 - 240d = -4$$

By subtracting $\quad - 232d = 85$

$$d = \frac{58}{-232}$$

$$d = -\frac{1}{4}$$

Putting $d = -\frac{1}{4}$ in equation (i) we get

$$4a_1 + 4 \times (\frac{-1}{4}) = 31$$

$$4a_1 - 1 = 31$$

$$4a_1 = 31 + 1$$

$$4a_1 = 32$$

$$a_1 = \frac{32}{4}$$

$$a_1 = 8$$

But, $a_n = \frac{-13}{2}$

$$a_1 + (n-1)\,d = \frac{-13}{2}$$

⇨ $8 + (n-1) \times (\frac{-1)}{4} = \frac{-13}{2}$

⇨ $(n-1) \times (\frac{-1}{4}) = \frac{-13}{4} - 8$

⇨ $(n-1) \times (\frac{-1}{4}) = \frac{-13-16}{4}$

⇨ $(n-1) \times (\frac{-1}{4}) = \frac{-29}{2}$

⇨ $(n-1) = \frac{-29}{2} \times (-4)$

⇨ $n-1 = 58$

⇨ $n = 58+1$

⇨ $n = 59$

Hence, the first term is 8 and the last term is 59.

6. Find the middle term of the AP 6, 13, 20, …. 216.

Solution: the given AP is 6, 13, 20, …. 216.

Here the first term, $a_1 = 6$

And the common difference, $d = 7$

Let there be n terms in the given AP

Then, $a_n = 216$

⇨ $a_1 + (n-1)\,d = 216$

⇨ $6 + (n-1)\,d = 216$

⇨ $(n-1) \times 7 = 216 - 6$

⇨ $(n-1) \times 7 = 210$

⇨ $n - 1 = \frac{210}{7}$

⇨ $n - 1 = 30$

⇨ $n = 30 + 1$

⇨ $n = 31$, which is odd.

∴ The middle term is $(\frac{n+1}{2})^{th}$.

i.e., $(\frac{31+1}{2})^{th} = (\frac{32}{2})^{th}$ or, 16^{th}.

Hence the middle term is a_{16} given by:

$$a_{16} = a_1 + 15d$$
$$= 6 + 15 \times 7$$
$$= 6 + 105$$
$$= 111$$

∴ The middle term of the AP is 111.

7. Find the sum of the following series: 72+70+68+…..+40

   Solution:

   Here $a_1 = 72$, $a_2 = 70$, $a_3 = 68$ and $a_n = 40$

   ∴ $a_3 - a_2 = 68-70 = -2$

   $a_2 - a_1 = 70-72 = -2$

   Since, $a_3 - a_2 = a_2 - a_1 = -2$

Therefore, the given series is an arithmetic series with first term $a_1 = 72$ and common difference, $d = -2$

Now, $a_n = 40$

⇨ $a_1 + (n-1)\,d = 40$

⇨ $72 + (n-1)\,(-2) = 40$

⇨ $(n-1) \times (-2) = 40 - 72$

⇨ $(n-1) \times (-2) = -32$

⇨ $(n - 1) = \frac{-32}{-2}$

⇨ $(n-1) = 16$

⇨ $n = 16 + 1$

⇨ $n = 17$

Using the formula:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

⇨ $S_{17} = \frac{17}{2}(72 + 40)$

⇨ $S_{17} = \frac{17}{2} \times 112$

⇨ $S_{17} = 17 \times 56$

⇨ $S_{17} = 952$

∴ The sum of the series 72+70+68+…..+40 is 952.

8. How many terms of the sequence 18, 16, 14,…..should be taken so that the series is 0?

   Solution: Given sequence is 18, 16, 14,…..

   Here $a_1 = 18$, $a_2 = 16$, $a_3 = 14$

Then $a_3 - a_2 = 14 - 16 = -2$ and $a_2 - a_1 = 16 - 18 = -2$

Since $a_3 - a_2 = a_2 - a_1 = -2$

The given sequence is arithmetic progression.

With first term, $a_1 = 18$ and common difference, $d = -2$

Let there be n numbers of terms that make the sum 0.

$$S_n = 0$$

$$\Rightarrow \frac{n}{2}(a_1 + a_n) = 0$$

$$\Rightarrow \frac{n}{2}\{a_1 + a_1 + (n-1)\ d\} = 0$$

$$\Rightarrow \frac{n}{2}\{2a_1 + (n-1)\ d\} = 0$$

$$\Rightarrow \frac{n}{2}\{2 \times 18 + (n-1)\ (-2)\} = 0$$

$$\Rightarrow n\ (36 - 2n + 2) = 2 \times 0$$

$$\Rightarrow n\ (38 - 2n) = 0$$

$$\Rightarrow n\ (38 - 2n) = 0$$

$$\Rightarrow 2n\ (19 - n) = 0$$

Either, $2n = 0$

$\Rightarrow n = \frac{0}{2}$

$\Rightarrow$ n = 0, is rejected as the number of terms cannot be zero

or, $19 - n = 0$

$\Rightarrow -n = -19$

$\Rightarrow n = 19$

Thus, there are 19th terms in the sequence that make the sum equal to zero.

9. (i) Find the sum of the first n natural numbers.
   (ii) Find the sum of the first 110 natural numbers.
   Solution: The given series is 1+2+3+4+…….+ n is an arithmetic series with

First term, $a_1 = 1$

And common difference, $d = 1$

Let there be n numbers of terms in the given series

$$\therefore \quad a_n = n$$

$$\Rightarrow \text{nth term} = n$$

Using the formula,

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{n}{2}(1 + n)$$

Hence the sum of the first n natural numbers is $\frac{n}{2}(1 + n)$

(ii) Here, n = 100

Sum of the first n natural numbers = $\frac{n(1 + n)}{2}$

∴ Sum first 100 natural numbers = $\frac{100(1 + 100)}{2}$

$= 50 \times 101$

$= 5050.$

10. If the sum of n terms of an AP is $(pn + qn^2)$ where p and q are constants, find the common difference.

Solution: Given sum of n terms of an AP is $(pn + qn^2)$

⇨ $S_n = (pn + qn^2)$

Let a_1, a_2 be the first and second term of an AP and d be the common difference.

Now, $S_n = (pn + qn^2)$

⇨ $S_1 = p + q$ (when n= 1)

⇨ $a_1 = p + q$ ....(i) [When n is 1 $S_1 = a_1$]

And, $S_2 = p \times 2 + q \times 2^2$

⇨ $a_1 + a_2 = 2p + 4q$ [∵ S_2 is the sum of the first and the 2nd term]

⇨ $(p + q) + a_2 = 2p + 4q$ [using (i)}

⇨ $a_2 = 2p + 4q - (p + q)$

⇨ $a_2 = p + 3q$

But, Common difference, $d = a_2 - a_1$

$= p + 3q - (p + q)$

$= p + 3q - p - q$

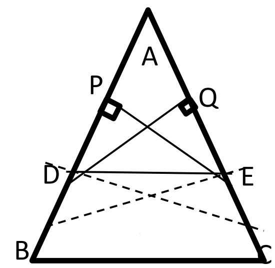
⇨ $d = 2q$

Hence, the common difference of the AP is 2q.

GEOMETRY

11. State and prove basic proportionality theorem.

Statement: If a line is drawn parallel to one side of a triangle intersecting the other two sides, in distinct point, then the other two sides are divided in the same ratio.



Given: ΔABC in which DE//BC

To prove: $\frac{AD}{BD} = \frac{AE}{EC}$

Construction: Join BE and CD

Draw EP⊥AB and DQ ⊥ AC

Proof: In ΔADE and BDE

$\frac{ar\,(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}\times AD\times PE}{\frac{1}{2}\times BD\times PE}$ [∵ ar right Δ = $\frac{1}{2}$ × b × h]

A

$\frac{ar\,(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{BD}$ (i)

Similarly, $\frac{ar\,(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2}\times AE\times DQ}{\frac{1}{2}\times EC\times DQ}$

⇨ $\frac{ar\,(\Delta ADE)}{ar(\Delta CDE)} = \frac{AE}{EC}$ (ii)

But Δ BDE and ΔCDE are on the same base DE and between the same parallels DE and BC

∴ ar(ΔBDE) = ar (ΔCDE) (iii)

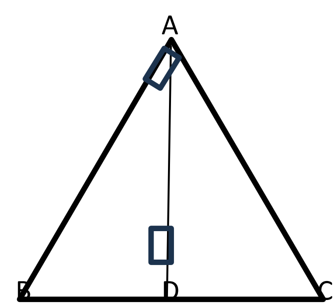
From equation (i), (ii) and (iii), we have

$\frac{AD}{BD} = \frac{AE}{EC}$

Hence, Proved.

12. Prove that in a right-angled Triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given: A right triangle ABC, right angled at B



To prove: $AC^2 = AB^2 + BC^2$

Construction: BD⊥AC is drawn

Proof: In ΔABD and ΔABC

∠ADB = ∠ABC (each 90^0)

∠A = ∠A (Common angles)

∴ Δ ADB ~ Δ ABC (AA Similarity)

⇨ $\frac{AD}{AB} = \frac{AB}{AC}$ [Sides are proportional]

⇨ AB^2 = AD × AC ……. (i)

Again, in ΔBDC and ΔABC

∠BDC = ∠ABC (each 90^0)

∠C = ∠C (Common angles)

$\Delta BDC \sim \Delta ABC$

⇨ $\frac{BC}{AC} = \frac{DC}{BC}$ (Sides are proportional)

⇨ $BC^2 = DC \times AC$ ………… (ii)

Adding (i) and (ii) we get

$AB^2 + BC^2 = AD \times AC + DC \times AC$

$= AC \times (AD + DC)$

$= AC \times AC = AC^2$

Hence, $AC^2 = AB^2 + BC^2$

13. Show that the ratio of the perimeters of the two similar triangles is the same as the ratio of their corresponding sides.

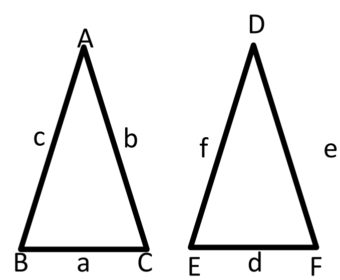
(i) The perimeter of two similar triangles is 25cm and 15cm respectively. If one side of first triangle is 9cm, what is the corresponding side of the triangles?

Let ΔABC and ΔDEF be similar

Let BC = a, CA = b, AB = c

And EF = d, FD = e, DE = f

$\because \Delta ABC \sim \Delta DEF$



⇨ $\frac{BC}{EF} = \frac{CA}{FD} = \frac{AB}{DE}$ (Sides are proportion)

⇨ $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$=k (say) (i)

⇨ $\boldsymbol{a = dk, b = ek, c = f}$

⇨ $\therefore \frac{\text{perimeter of} \Delta ABC}{\text{perimeter of } \Delta DEF} = \frac{a+b+c}{d+e+f}$

$= \frac{dk+ek+fk}{d+e+f}$

$= \frac{k(d+e+f)}{d+e+f}$

$= \boldsymbol{k}$

$\therefore \frac{\text{perimeter } \Delta ABC}{\text{perimeter of } \Delta DEF} = \frac{a}{d} = \frac{b}{e} = \frac{c}{f}$

$= \frac{BC}{EF} = \frac{CA}{FD} = \frac{AB}{DE}$

(ii) Let $\Delta ABC \sim \Delta DEF$ such that BC = 9cm

perimeter of ΔABC = 25cm

Perimeter of ΔDEF = 15cm

$\because \Delta ABC \sim \Delta DEF$

$\therefore \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{BC}{EF}$

⇨ $\frac{25}{15} = \frac{9}{EF}$

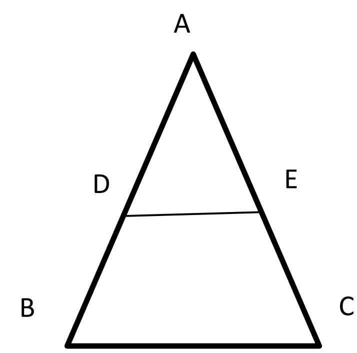
⇨ $EF = \frac{9 \times 15}{25}$

⇨ $EF = \frac{27}{5}$

⇨ EF = 5.4 cm

14. In the figure, if DE//BC, AD = 4x – 3, AE = 8x -7, BD = 3x – 1 and CE = 5x – 3. Find the value of x

Solution: In ΔABC, DE//BC



$\therefore \frac{AD}{BD} = \frac{AE}{EC}$ [Basic proportionality theorem]

⇨ $\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$

⇨ $(4x - 3)(5x - 3) = (8x - 7)(3x - 1)$

⇨ $4x(5x - 3) - 3(5x - 3) = 8x(3x - 1) - 7(3x - 1)$

⇨ $20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$

⇨ $20x^2 - 27x + 9 = 24x^2 - 29x + 7$

⇨ $20x^2 - 24x^2 - 27x + 29x + 9 - 7 = 0$

⇨ $-4x^2 + 2x^2 + 2x + 2 = 0$

⇨ $-2(2x^2 - x - 1) = 0$

⇨ $2x^2 - x - 1 = \frac{0}{-2}$

⇨ $2x^2 - x - 1 = 0$

⇨ $2x^2 - 2x + x - 1 = 0$

⇨ $2x(x - 1) + 1(x - 1) = 0$

⇨ $(x - 1)(2x + 1) = 0$

Either, x -1 =0 => x = 1

Or, 2x + 1 = 0 => x = - ½

$x = -\frac{1}{2}$, is rejected as the length cannot be negative.

Hence x = 1 unit

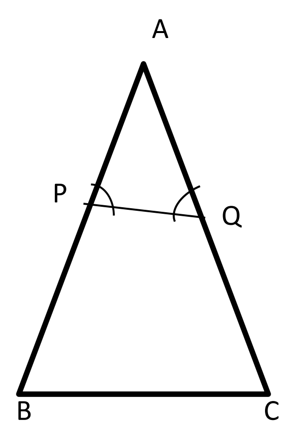
15. In the fig, ∠APQ = ∠B, Prove that ΔAPQ ~ ΔABC. If AP = 3.8cm, AQ = 3.6cm, BQ=2.1cm and BC = 4.2cm, Find PQ.

Solution: In ΔAPQ and ΔABC

∠APQ = ∠B (Given)

∠A= ∠A (Common angles)

∴ Δ APQ ~ Δ ABC (AA similarity)



⇨ $\frac{AP}{AB} = \frac{PQ}{BC}$ (sides are proportional)

⇨ $\frac{AP}{AQ+BQ} = \frac{PQ}{BC}$ (∵ AB = AQ + BQ)

⇨ $\frac{3.8}{3.6+2.1} = \frac{PQ}{BC}$

⇨ $\frac{3.8}{5.7} = \frac{PQ}{BC}$

⇨ $PQ = \frac{3.8 \times 4.2}{5.7}$

⇨ $PQ = 2 \times 1.4$

⇨ $PQ = 2.8$

Hence, PQ = 2.8cm

16. Prove that the ratio of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Given: Two triangles ABC and DEF, such that ΔABC ~ ΔDEF

To Prove: $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Construction: Draw AP⊥BC and PQ⊥EF

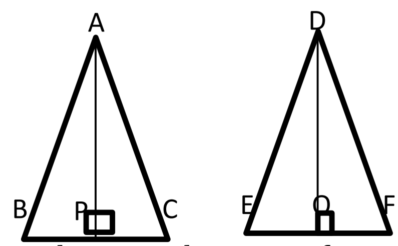
Proof: ∵ ΔABC ~ ΔDEF

$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ (i) (sides are proportional)

⇨ $\frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$ (ii) (by squaring)

Now, $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{\frac{1}{2} \times BC \times AP}{\frac{1}{2} \times EF \times DQ}$

$= \frac{BC}{EF} \times \frac{AP}{DQ}$ (iii)



Also, $\frac{Ap}{DQ} = \frac{BC}{EF}$ (iv) [∵ in similar triangles ratio of corresponding sides is the same as ratio of corresponding altitudes]

From (iii) and (iv) we get

$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC}{EF} \times \frac{BC}{EF}$

$= \frac{BC^2}{EF^2}$

$= \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$ [using equation (ii)]

Hence, $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

17. In a triangle, if the square of one side is equal to the sum of the square of the remaining two sides, prove that the triangle is a right-angled triangle.

Using the above result, do the following

Determine whether a triangle having sides a = 5cm, b = 12cm and c = 13cm is a right-angled triangle or not.

Given: A triangle ABC such that $AB^2 + BC^2 = AC^2$

To prove: $\angle ABC = 90^0$

Construction: Construct a right-angled triangle DEF, right angled at E, such that DE = AB and EF = BC

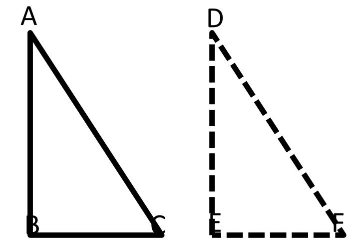
Proof: in rt ΔDEF

$DE^2 + EF^2 = DF^2$ [Pythagoras theorem]

⇨ $AB^2 + BC^2 = DF^2$ [∵ DE = AB and B EF = BC]

⇨ $AC^2 = DF^2$ [∵ $AB^2 + BC^2 = DF^2$]

⇨ AC = DF



In ΔABC and ΔDEF

AB = DE (given)

BC = EF (given)

AC = DF (proved above)

∴ ΔABC ≅ ΔDEF [By sss congruence criteria)

⇨ ∠ABC = ∠DEF [c.p.c.t.c]

But, ∠ABC = 90^0

Given: a =5cm, b = 12cm and c = 13cm

⇨ $a^2 = 25$, $b^2 = 144$cm2 and $c^2 = 164cm^2$

Now, $a^2 + b^2 = 25cm^2 + 144cm^2 = 169cm^2 = c^2$

∴ $c^2 = a^2 + b^2$

By the converse of Pythagoras theorem.

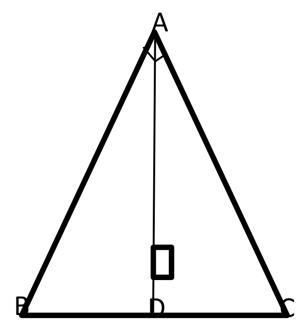
Hence, ΔABC is a right-angled triangle.

18. ΔABC is right-angled at A and AD⊥BC. If BC = 13cm and AC = 5cm. Find the ratio of the areas of ΔABC and ΔADC.

Solution: Given ΔABC, ∠A = 90^0, AD⊥BC

Also given BC = 13cm and AC = 5cm

To find: $\frac{ar(\Delta ABC)}{ar(\Delta ADC)}$



Proof: In ΔABC and ΔADC

∠BAC = ∠ADC [Each is 90^0]

∠C = ∠C [common angles]

∴ ΔABC ~ ΔDAC [AA similarity]

⇨ $\frac{ar(\Delta ABC)}{ar(\Delta DAC)} = \frac{BC^2}{AC^2}$ [∵ the ratio of the areas of two similar triangle is equal to the ratio of the squares of their corresponding sides]

⇨ $\frac{ar(\Delta ABC)}{ar(\Delta DAC)} = \frac{13^2}{5^2}$

$= \frac{169}{25}$

Hence, ar(ΔABC): ar(ΔADC) = 169: 25

19. (i) A man goes 15m due east and then 8m due north. How far is he from the starting point?

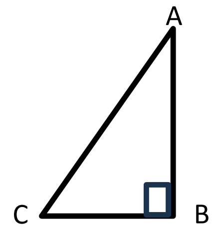
(ii) ΔABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$,

Prove that ΔABC is a right angled triangled

Solution (i): Let C be the starting point

Let A be the ending point

In rt ΔABC



$AC^2 = AB^2 + BC^2$ (using Pythagoras theorem)

$= (8m)^2 + (15m)^2$

$= 64m^2 + 225m^2$

$AC^2 = 289m^2$

$\sqrt{AC^2} = \sqrt{289m^2}$

AC = 17

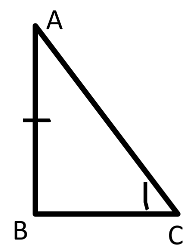
Hence, AC = 17m.

Solution (ii) Given, AC = BC

And, $AB^2 = 2AC^2$

To prove: ΔABC is right angle Δ

Proof: In ΔABC



$AB^2 = 2\ AC^2$

$AB^2 = AC^2 + AC^2$

$AB^2 = BC^2 + AC^2$ [$\because$ AC = BC]

By the converse of Pythagoras theorem.

$\therefore$ ΔABC is a right-angle triangle.

20. In Rhombus of side 10cm, one of the diagonals is 12cm long. Find the length of second diagonal.

Solution: Let ABCD be the rhombus where diagonals AC and BD intersect at O.

AB = 10cm and AC = 12cm

Let BD =2x cm

Since diagonals of a rhombus bisect each other at right angle.

$\therefore$ $AO = \frac{1}{2} AC = \frac{1}{2} \times 12 = 6cm$

$OB = \frac{1}{2} BD = \frac{1}{2} \times 2x = x cm$

Also, $\angle AOB = 90^0$

Now, in right angle ΔAOB

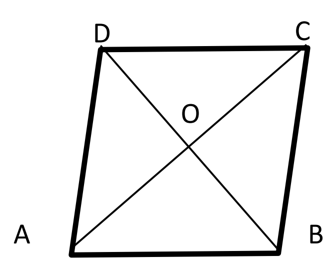
$AB^2 = AO^2 + OB^2$ [using Pythagoras theorem]

⇨ $10^2 = 6^2 + x^2$

⇨ $100 = 36 + x^2$

⇨ $100 - 36 = x^2$

⇨ $64 = x^2$



⇨ $\sqrt{64} = x$

⇨ $8 = x$

$\therefore x = 8cm$

$\therefore BD = 2x = 2\times8 = 16cm$

Hence the length of another diagonal is 16cm.

TRIGONOMETRY

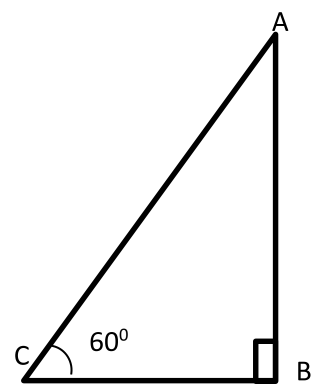
21. A kite is flying at a height of 75 metres from the level ground, attached to a string inclined at 60^0 to the horizontal. Find the length of the string. (use $\sqrt{3} = 1.732$)

Solution: Height of a kite from the ground, AB = 75m

Length of the string be AC

Angle of elevation, $\Theta = 60^0$

Now, In right ΔABC



$\sin\Theta = \frac{AB}{AC}$

⇨ $\text{Sin}60^0 = \frac{75}{AC}$

⇨ $\frac{\sqrt{3}}{2} = \frac{75}{AC}$

⇨ $AC = \frac{75\times2}{\sqrt{3}}$

⇨ $AC = \frac{150}{\sqrt{3}}$

⇨ $AC = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

⇨ $AC = \frac{150\sqrt{3}}{3}$

⇨ $AC = \frac{150\times1.732}{3}$

⇨ $AC = 86.6$

⇨ Hence the length of the string is 86.6m

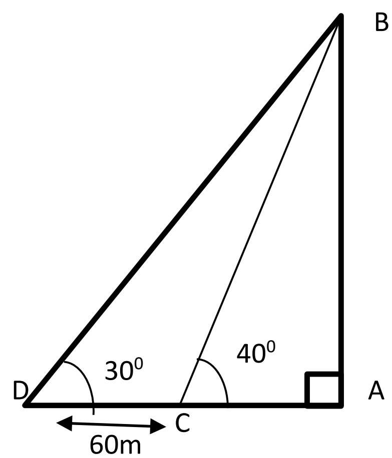
22. The shadow of a vertical tower is found to be 60m longer on the level ground when the sun's altitude is 30^0 than when it is 45^0. Find the height of the tower. (use $\sqrt{3}$ = 1.732)

Solution: Let AB be the height of the tower

AC be the length of the shadow when sun's altitude is 45^0

AD = (AC + 60) m, the length of the shadow when sun's altitude is 30^0

In right ΔBAC, ∠ACB = 45^0



$$\frac{AB}{AC} = \tan \angle ACB$$

⇨ $\frac{AB}{AC} = \tan 45^0$

⇨ $\frac{AB}{AC} = 1$

⇨ AB = AC

In right ΔBAD, ∠ADB = 30^0

$$\frac{AB}{AC} = \tan \angle ADB$$

⇨ $\frac{AB}{AD} = \tan \angle ADB$

⇨ $\frac{AB}{AB+CD} = \tan 30^0$

⇨ $\frac{AB}{AB+60} = \frac{1}{\sqrt{3}}$ [∵ AB = AC]

⇨ $AB \times \sqrt{3} = AB + 60$

⇨ $AB\sqrt{3} - AB = 60$

⇨ $AB(\sqrt{3} - 1) = 60$

⇨ $AB = \frac{60}{\sqrt{3}-1}$

⇨ $AB = \frac{60}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

⇨ $AB = \frac{60\,(\sqrt{3}+1)}{(\sqrt{3})^2 - 1^2}$

⇨ $AB = \frac{60\,(1.732+1)}{3-1}$

⇨ $AB = \frac{60 \times 2.732}{2}$

⇨ $AB = 30 \times 2.732$

⇨ AB = 81.96m

Hence the height of the tower is 81.96m

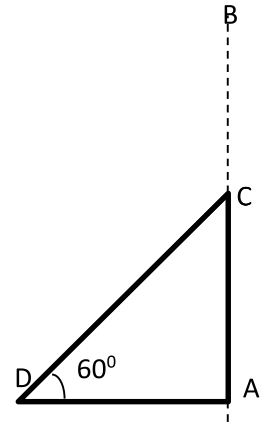
23. A vertically straight tree 15m high is broken by the wind in such a way that its top touches the ground and makes an angle 60^0 with the ground. At what height from the ground did the tree break? (use $\sqrt{3} = 1.732$)

Solution: Let AB be the tree, broken at point C such that AC = xm

Let CB take the position CD be the broken part of the tree.

Then, CD = CB = (15 – x) m

And $\angle ADC = 60^0$



In right ΔDAC, we have

$\frac{AC}{CD} = \sin 60^0$

⇨ $\frac{x}{15-x} = \frac{\sqrt{3}}{2}$

⇨ $2x = \sqrt{3}(15 - x)$

⇨ $2x = 15\sqrt{3} - \sqrt{3}x$

⇨ $2x + \sqrt{3}x = 15\sqrt{3}$

⇨ $x(2 + \sqrt{3}) = 15\sqrt{3}$

⇨ $x = \frac{15\sqrt{3}}{\sqrt{3}+2}$

⇨ $x = \frac{15\sqrt{3}}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$

⇨ $x = \frac{15\sqrt{3}(\sqrt{3}-2}{\sqrt{3}^2-2^2}$ [∵ (a-b) (a+b) = $a^2 + b^2$]

⇨ $x = \frac{15\times3-30\sqrt{3}}{3-4}$

⇨ $x = \frac{45-30\times1.73}{-1}$

⇨ $x = \frac{-6.9}{-1}$

⇨ x = 6.9

Thus, the tree broke at a height of 6.9m from the ground.

24. A vertically tower on a horizontal plane and is surmounted by a vertical flag-staff of height 5m, from a point on the plane the angles of elevation of the bottom and the top of the flag-staff are 30^0 and 60^0. Find the height of the tower.

Solution: let AB be the height of the tower and BC be the flag-staff

Let 0 be the position of the observer.

Then, $\angle AOB = 30^0$, $\angle AOC = 60^0$ and BC = 5m

In right Δ0AB, we have

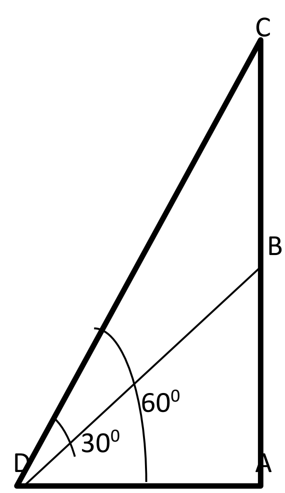
$\frac{AB}{OA} = \tan \angle AOB$

⇨ $\frac{AB}{OA} = \tan 30^0$

⇨ $\frac{AB}{0A} = \frac{1}{\sqrt{3}}$

⇨ $OA = AB\sqrt{3}$ (i)

In right ΔOAC, we have



$\frac{AC}{OA} = \tan\angle AOC$

⇨ $\frac{AB+5}{AB\sqrt{3}} = \tan 60^0$

⇨ $\frac{AB+5}{AB\sqrt{3}} = \sqrt{3}$

⇨ $AB + 5 = AB\sqrt{3} \times \sqrt{3}$

⇨ $AB + 5 = 3AB$

⇨ $AB - 3AB = -5$

⇨ $-2AB = -5$

⇨ $AB = \frac{-5}{-2}$

⇨ $AB = 2.5$

Hence, the height of the tower is 2.5m.

25. From the top of the building 15m high, the angle of elevation of the top of a tower is found to be 30^0. From the bottom of the same building, the angle elevation of the top of the tower is found to be 60^0. Find the height of the tower and the distance between the tower and the building. (use $\sqrt{3} = 1.732$)

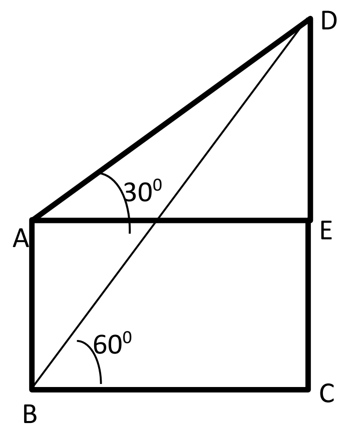
Solution: Let AB = 15m be the height of the building.

Let CD= h m be the height of the tower

∴ ED = (h -15) m

$\angle EAD = 30^0$ and $\angle CBD = 60^0$

In right ΔAED, we have



$\frac{ED}{AE} = \tan \angle EAD$

⇨ $\frac{h-15}{AE} = \tan 30^0$

⇨ $\frac{h-15}{AE} = \frac{1}{\sqrt{3}}$

⇨ $AE = (h - 15)\sqrt{3}$ (i)

Again, in right ΔBCD, we have

$\frac{CD}{BC} = \tan \angle CBD$

⇨ $\frac{h}{AE} = \tan 60^0$ [∵ BC = AE]

⇨ $\frac{h}{(h-15)\sqrt{3}} = \sqrt{3}$

⇨ $h = (h - 15)\sqrt{3} \times \sqrt{3}$

⇨ $h = (h - 15) \times 3$

⇨ $h = 3h - 45$

⇨ $h - 3h = -45$

⇨ $-2h = -45$

⇨ $h = \frac{-45}{-2}$

⇨ $h = 22.5$

∴ The height of the tower is 22.5m

Putting h=22.5 in equation (i) we get

AE = (22.5 -15) $\sqrt{3}$

AE = 7.5 × $\sqrt{3}$

AE = 7.5 × 1.732

AE = 12.99m

Therefore, the distance between the building and the tower is 12.99m

26. The angle of elevation of the top of a tree from a point A on the ground is 60^0. On walking 20m away from the its base, to a point B, the angle of elevation changes to 30^0. Find the height of the tree. (use $\sqrt{3} = 1.732$)

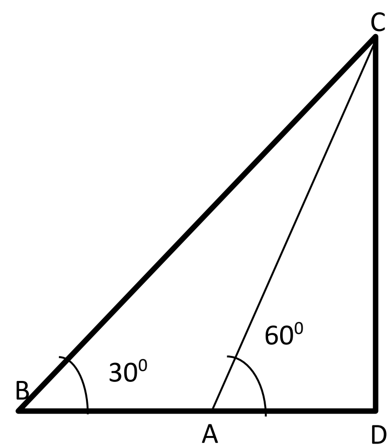
Solution: Let CD be the tree of height h metres and let AD = x metres

Then,

AB = 20m, ∠DAC = 60^0

And ∠ABC = 30^0

Now, in right ΔADC



$\frac{CD}{AD}$ = tan60^0

⇨ $\frac{h}{x} = \sqrt{3}$

⇨ h = x$\sqrt{3}$ (i)

In right ΔBDC

$\frac{CD}{AD}$ = tan30^0

⇨ $\frac{h}{AB+AD} = \frac{1}{\sqrt{3}}$

⇨ $\frac{h}{20+x} = \frac{1}{\sqrt{3}}$

⇨ h = $\frac{20+x}{\sqrt{3}}$

⇨ x$\sqrt{3}$ = $\frac{20+x}{\sqrt{3}}$ [using equation (i)]

⇨ x$\sqrt{3}$ × $\sqrt{3}$ = 20 + x

⇨ 3x = 20 + x

⇨ 3x -x = 20

⇨ 2x = 20

⇨ x = $\frac{20}{2}$

⇨ x = 10

Putting x =10 in equation (i) we get

h = 10 × $\sqrt{3}$

h = 10 × 1.732

$h = 17.32$

Hence, the height of the tree is 17.32m

27. From the top of the building 60m high, the angle of depression of the top and the bottom of a tower are observed to be 30^0 and 60^0. Find the height of the tower.

Solution: Let AB = 60m be the building

Let CD = hm be the tower

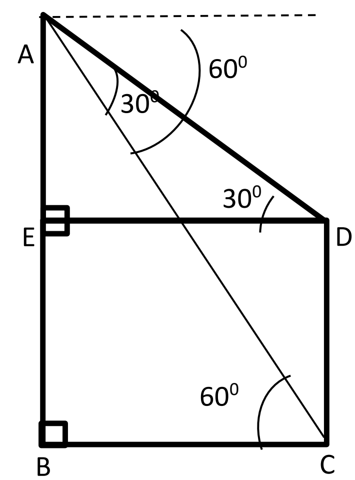
Then, AE = (60 – h)m , BE = CD = 30^0, ∠ACB = 60^0

Now, In right ΔAED, we have

$$\frac{AE}{ED} = \tan 30^0$$

⇨ $$\frac{60-h}{x} = \frac{1}{\sqrt{3}}$$

⇨ $$x = (60 - h)\sqrt{3} \qquad \text{(i)}$$



Again, in right ΔABC, we have

$$\frac{AB}{BC} = \tan 60^0$$

⇨ $$\frac{60}{x} = \sqrt{3}$$

⇨ $$x = \frac{60}{\sqrt{3}} \qquad \text{(ii)}$$

Substituting (ii) in (i) we get

$$\frac{60}{\sqrt{3}} = (60 - h)\sqrt{3} \times \sqrt{3}$$

⇨ $60 = (60 - h) \times 3$

⇨ $\frac{60}{3} = 60 - h$

⇨ $20 = 60 - h$

⇨ $20 - 60 = -h$

⇨ $-40 = -h$

⇨ $40 = h$

Or, h = 40

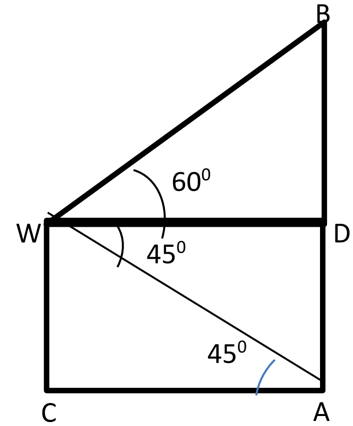
Hence the height of the tower is 40m

28. From a window 60m high above the ground of a house in a street. The Angle of elevation and depression of the top and the foot of another house on the opposite sides of the street are 60^0 and 45^0 respectively. Show that the height of the opposite house is $60(\sqrt{3} + 1)$m

Solution: Let AB be one house of height hm

Let w be the window of the other house CW, such that CW = 60m

Then, AD = CW = 60m. ∠BWD = 60^0



∠CAW = ∠AWD = 45^0

Let WD = CA = x m and BD = (h-60)m

Now, in right ΔWCA, we have

$$\frac{CW}{AC} = \tan 45^0$$

⇨ $\frac{60}{x} = 1$

⇨ x = 60m

⇨ WD = CA = 60m

Again, in right ΔBDW, we have

$$\frac{BD}{WD} = \tan 60^0$$

⇨ $\frac{h-60}{60} = \sqrt{3}$

⇨ $h - 60 = 60\sqrt{3}$

⇨ $h = 60\sqrt{3} + 60$

⇨ $h = 60(\sqrt{3} + 1)$

Hence the height of the opposite house is $60(\sqrt{3}+1)$ m

29. A straight highway leads to the foot of the tower of height 50m. from the top of the tower, the angle of depression of two cars standing on the highway are 30^0 and 60^0 respectively. What is the distance between two cars and how far is each car from the tower?

Solution: Let AB be the tower of height 50m

Let point D be the position of first car

Let point C be the position of the second car

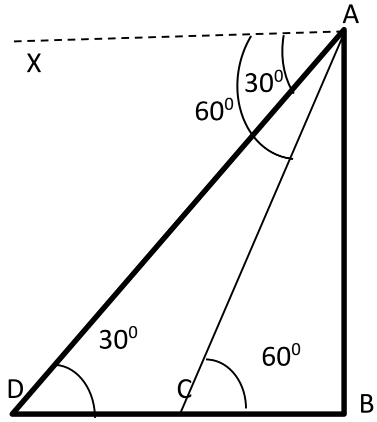
Then, CD=xm, distance between the two cars,

BD = (x+y) m distance of the first car from the tower

And BC = ym distance of the second car from the tower

Also $\angle XAD = \angle ADB = 30^0$ and $\angle XAC = \angle ACB = 60^0$

Now, in right ΔABC, we have



$$\frac{AB}{BD} = \tan 30^0$$

⇨ $$\frac{50}{x+y} = \frac{1}{\sqrt{3}}$$

⇨ $$x + y = 50\sqrt{3}$$

⇨ $$x + y = 50 \times 1.732$$

⇨ $$x + y = 86.5 \qquad \text{(i)}$$

Again, In right ΔABC

$$\frac{AB}{BC} = \tan 60^0$$

⇨ $$\frac{50}{y} = \frac{50}{\sqrt{3}}$$

⇨ $$y = \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

⇨ $$y = \frac{50\sqrt{3}}{3}$$

⇨ $$y = \frac{50 \times 1.732}{3}$$

⇨ $$y = \frac{86.5}{3}$$

⇨ $$y = 28.83$$

Putting y = 28.83 in equation (i) we get

$$x + 28.83 = 86.5$$

⇨ $$x = 86.5 - 2.83$$

⇨ $$x = 57.67m$$

Hence the distance between the two cars is x = 57.67m

Distance of first car from the tower is (x + y) = 86.5m

Distance of the second car from the tower is (y) = 28.83m

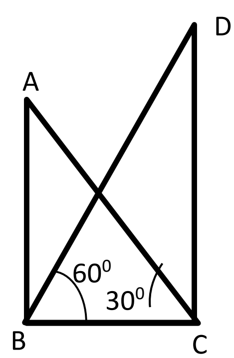
30. An angle of elevation of the top of a hill from the foot of a tower is 60^0 and the angle of elevation of the top of the top of the tower from the foot of the hill is 30^0. If the tower is 50m high. Find the height of the hill.

Solution: Let AB be the tower of height 50m

Let CD be the hill of height h metres.

Given $\angle CBD = 60^0$ and $\angle ACB = 30^0$

In right ΔABC, we have



$$\frac{AB}{BC} = \tan 30^0$$

⇨ $$\frac{50}{BC} = \frac{1}{\sqrt{3}}$$

⇨ $BC = 50\sqrt{3}$ (i)

Again, in right ΔBCD, we have

$$\frac{CD}{BC} = \tan 60^0$$

⇨ $\frac{h}{50\sqrt{3}} = \sqrt{3}$ [∵ $BC = 50\sqrt{3}$]

⇨ $h = 50\sqrt{3} \times \sqrt{3}$

⇨ $h = 50 \times 3$

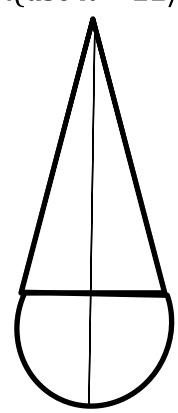
⇨ $h = 150$

Hence, the height of the hill is 150m

MENSURATION

31. A toy is in the form of a cone mounted on a hemisphere of radius 3.5 cm. The total height of the toy is 15.5 cm. Find the total surface area of the toy.(use π = 22/7)

Solution:



Height of the toy (H) = 15.5cm

Radius of the hemisphere (r) -=3.5 cm

Height of the cone (h)= H – r

=15.5 - 3.5

=12 cm

height of the cone, $l = \sqrt{r^2 + r^2}$

$= \sqrt{(12)^2 + (35^2)}$

$= \sqrt{144 + 12 - 25}$

$= \sqrt{156.25}$

=12.5 cm

TSA of the toy = CSA of the cone + CSA of hemisphere

$= \pi rl + 2\pi r^2$

$= \pi r(l+2r)$

$= \frac{22}{7} \times 3.5\ (12.5+2\times 2x3.5)$

$=22\times 0.5\times(12.5+7)$

$=11\times 19.5$

$=214.5\ cm^2$

Hence, the total surface area of the toy is $214.5cm^2$

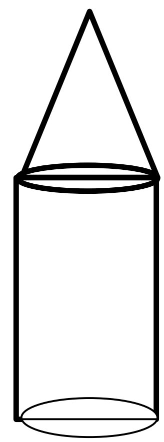
32. A military tent of height 8.25m is in the form of a right circular cylinder of base diameter 30m and height 5.5m surrounded by a right circular cone of same base radius. Find the length of canvas used in making the tent, if the breadth of canvas is 1.5m.

Solution:

Radius of the cone = Radius of cylinder = r

So, $r = \frac{1}{2} \times 30m$ (∵ radius = ½ x diameter)

=> $r = 15m$



Height of the cylinder, H = 5.5m

Height of the cone, h = Total - H

$= 8.25 - 5.5$

$= 2.75m$

Slant height of the cone, $l = \sqrt{r^2 + h^2}$

$= \sqrt{(15)^2 + (2.75)^2}$

$= \sqrt{225 + 7.5625}$

$l = \sqrt{232.5625}$

$l = 15.25$ m

Area of the canvas = CSA of the cone + CSA of the cylinder

$= \pi rl + 2\pi rH$

$= \pi r\ (l + 2H)$

$= \frac{22}{7} \times 15 \times (15.25 + 2 X 5.5)$

$= \frac{22}{7} \times 15 \times (15.25 + 11)$

$= \frac{22}{7} \times 15 \times 26.25$

$= \frac{8662.5}{7} m^2$

=> length × breadth = 8662.5/7

=> length × 1.5 = 8662.5/7

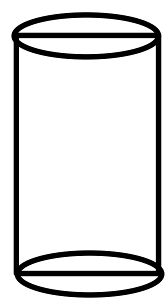
=> length = $\frac{8662.5}{7}$× 1.5

$= \frac{866.25}{7\times15} \times \frac{10}{10}$

= 825m

Hence, length of the canvas used in making the tent is 825m

33. A wooden article was made by scooping out a hemisphere from each end of a cylinder, as shown in the figure. If the height of the cylinder is 20cm and its base is of diameter 7cm, find the total surface area of the article.



Solution: Radius of the cylinder=Radius of the hemisphere=r

Hence, $r=\frac{1}{2}\times7$ [∵ $r=\frac{1}{2}\times$diameter]

$r=\frac{7}{2}$ cm

TSA of the article = 2×CSA of scooping hemisphere+ CSA of the Cylinder

$= 2\times2\pi r^2 + 2\pi rh$

$= 2\pi r(2r+h)$

$= 2\times\frac{22}{7}\times\frac{7}{2} \times (2\times\frac{7}{2} +20)$

= 22×27

$= 594cm^2$

Hence TSA of the article is $594cm^2$

34. A 5cm wide cloth is used to make a conical tent of base diameter 14cm and the height 24cm. Find the cost of the cloth used at the rate of Rs25 per meter.

Solution: Radius of the cone, $r = \frac{1}{2} \times 14m = 7m$

Height of the cone, h= 24m

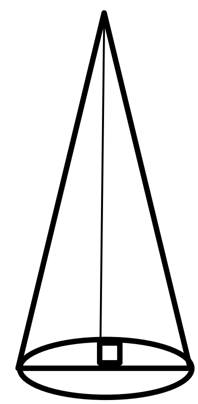
Slant height of the cone, $l = \sqrt{r^2 + h^2}$

$= \sqrt{(7)^2 + (24)^2}$

$= \sqrt{49 + 576}$

$= \sqrt{625}$

$= 25m$



∴ Area of the cloth = CSA of the cone

$= \pi rl$

$= \frac{22}{7} \times 7 \times 25$

$= 22 \times 25$

$= 550m^2$

⇨ length × breadth = $550m^2$

⇨ Length ×5 = 550

⇨ Length $= \frac{550}{5}$

$= 110m$

Hence, the length of the cloth is 110 m

Now, cost of 1m of cloth =Rs 25

Therefore, cost of 110m of cloth=Rs 25×110= Rs 2750.

35. . From a solid cylinder of height 12cm and base diameter 10cm, a conical cavity with the same height and diameter is carved out. Find the volume of the remaining solid (use π=22÷7).

Solution: since the conical cavity carved out of the solid cylinder have same height and same diameter.

Therefore, for Conical cavity and solid cylinder, diameter=10cm

Therefore Radius (r) = $\frac{10}{2}$=5cm

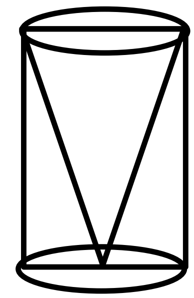
And Height (h) =12cm

Volume of the remaining solid= Volume of the solid cylinder-Volume of the conical cavity

$=\pi r^2h-\frac{1}{3}\pi r^2h$

$=\pi r^2h\ [1-\frac{1}{3}]$

$=\pi r^2h\times\frac{2}{3}$



$=\frac{22}{7}\times5\times5\times12\times\frac{2}{3}$

$=\frac{440}{7}$

$=628.57cm^3$

Hence, the volume of the remaining solid is $628.57cm^3$.

36. If the total surface of a solid hemisphere is $462cm^2$, find its volume (Take $\pi=22\div7$)
    Solution: Let 'r' be the radius of the hemisphere
    Given:
    TSA of a hemisphere $=462cm^2$

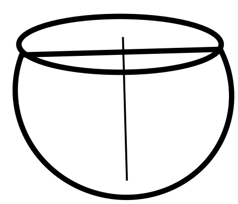
⇨ $3\pi r^2=462$

⇨ $3\times\frac{22}{7}\times r^2=462$

⇨ $r^2=462\times7\div(3\times22)$

⇨ $r^2=7^2$

⇨ $r=7cm$



∴ Volume of the hemisphere $=\frac{2}{3}\pi r^3$

$=\frac{2}{3}\times\frac{22}{7}\times7\times7\times7$

$=\frac{2156}{3}$

$=718.67cm^3$.

Hence the volume of hemisphere is 718.67 cm^3

37. A metal plate, which is 10cm thick, 9cm wide and 81cm long is melted and recast into a cube. Find the difference in the surface areas of the two solids.
    Solution: Surface area of metal plate=2(lb+bh+hl)

$=2(81\times9+9\times1+1\times81)$

$=2(729+9+81)$

$=2\times819$

$=1638cm^2$

Volume of the metal plate $=lbh$

$=81\times9\times1$

$=729cm^3$

Let the edge of the cube be 'a'

Since it melted and recast into a cube

∴ volume of the cube=volume of the metal plate

$a^3=729$

$a=\sqrt[3]{729}$

$a=9cm$

Surface area of the cube$=6a^2$

$=6\times9\times9$

$=486\ cm^2$

Hence, the difference in surface area of the two solids

$=(1638-486)\ cm^2$

$=1152cm^2$.

38. A metallic sphere of radii 6cm, 8cm and 10cm respectively are melted to form a single solid sphere. Find the radius of the resulting sphere.

Solution:

Volume of sphere of radius 6cm $=\frac{4}{3}\pi r^3$

$=\frac{4}{3}\pi(6)^3cm^3$

$=288\pi cm^3$

Volume of sphere of radius 8cm$=\frac{4}{3}\pi(8)^3cm^3$

$=\frac{2048\pi}{3}cm^3$

Volume of sphere of radius 10cm$=\frac{4}{3}\pi\times10\times10\times10cm^3$

$=\frac{4000\pi}{3}cm^3$

∴ total volume of the three sphere$=(288\pi+\frac{2048\pi}{3}+\frac{4000\pi}{3})\ cm^3$

$=(288\pi+\frac{6048\pi}{3})\ cm^3$

$=2304\pi cm^3$

Let R cm be the radius of the resulting sphere.

Since, the three small sphere is melted to for big sphere

$\therefore \frac{4}{3}\pi R^3=2304\pi$

$R^3 = \frac{2304\pi\times3}{4\times\pi}$

$R^3 = 1728$

$R = \sqrt[3]{1728} = 12cm$

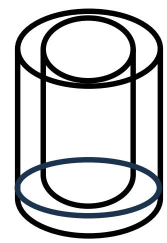
Hence, Radius of the resulting sphere is 12cm.

39. The difference between the outer and inner curved surface area of a hollow right circular cylinder 14cm long is $88cm^2$. If the volume of the metal used in making the cylinder is $176cm^3$, find the outer and inner diameters of the cylinder. (Use$\pi=22\div7$)

Solution:

Let r_1 be the radius of the inner cylinder

Let r_2 be the radius of the outer cylinder



Given: height of the cylinder, h=14cm

Also,

Given,

Outer CSA of cylinder- inner CSA of cylinder=$88cm^2$

$2\pi r_2h-2\pi r_1h = 88$

$2\pi h(r_2-r_1) = 88$

$r_2-r_1=\frac{88}{2\pi h}$

$=\frac{44\times7}{22\times14}$

$r_2-r_1 = 1$……… (i)

Also,

Volume of outer cylinder-volume of inner cylinder= $176cm^3$

⇨ $\pi r_2^2h - \pi r_1^2h = 176$

⇨ $\pi h(r_2^2-r_1^2) = 176$

⇨ $(r_2^2-r_1^2) = \frac{176}{\pi h}$

⇨ $(r_2^2-r_1^2)=\frac{176\times7}{22\times14}$

⇨ $(r_2^2-r_1^2) = 4$

⇨ $(r_2-r_1)(r_2+r_1) = 4$ $\{\because a^2-b^2=(a+b)(a-b)\}$

⇨ $1\times(r_2+r_1) = 4$(using (i)

⇨ $(r_2+r_1) = 4$ …….. (ii)

Adding (i) and (ii), we get,

$2r_2 = 5$

$\Rightarrow \quad r_2 = \frac{5}{2}$ cm

Putting $r_2 = \frac{5}{2}$ cm in (ii), we get,

$\frac{5}{2} + r_1 = 4$

- ⇨ $5 + 2r_1 = 8$
- ⇨ $2r_1 = 8 - 5$
- ⇨ $r_1 = \frac{3}{2}$ cm

Hence, Diameter of the outer cylinder = $2r_2$

$= 2 \times \frac{5}{2}$

$= 5$cm

And, Diameter of the inner cylinder $= 2r_1$

$= 2 \times \frac{3}{2}$

$= 3$ cm

40. A metallic sphere of radius 10.5cm is melted and then recast into small cones, each of radius 3.5cm and height 3cm. Find how many cones are obtained.

Solution:

Radius of a sphere, $R = 10.5$cm

Radius of the cone, $r = 3.5$cm

Height of the cone, $h = 3$cm

Now,

Volume of the sphere$= \frac{4}{3}\pi R^3$

$= \frac{4}{3}\pi \times (10.5)^3$

$= \frac{4}{3}\pi \times 10.5 \times 10.5 \times 10.5 cm^3$

Volume of one cone$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 3.5 \times 3.5 \times 3$

$= \pi \times 3.5 \times 3.5 cm^3$

Let x be the number of small cones

Since, a metallic sphere is melted and recast into small cones

∴ Volume of a sphere = number of small cones× volume of one cone

- ⇨ $\frac{4}{3}\pi\times10.5\times10.5\times10.5 = x\times\pi\times3.5\times3.5$
- ⇨ $\frac{4\pi\times10.5\times10.5\times10.5}{3\times\pi\times3.5\times3.5} = x$
- ⇨ $4\times3\times10.5 = x$
- ⇨ $x = 126$

∴ there are 126 small cones.

Sample Question Paper

(SSLC Examination 2024-25)

Mathematics

(Old Course)

by

Meghalaya Board of School Education (MBOSE)

A. Scheme of Theory Examination

Section	Type of Questions	Marks for Each Question	No. of questions to be attempted/ no. of questions given	Total Marks
Section-A	MCQs	1	30/30	1x30=30
Section-B	Very Short Answer Questions	2	6/9	2x6=12
Section-C	Short Answer Questions	3	6/9	3x6=18
Section-D	Long Answer Questions	5	4/7	5x4=20
Total Marks				**80**

Sample Question Paper

Mathematics (Old Course)
Class-X

Question Paper Code: XY

Time: 3 hours **Max Marks: 80 (Pass Marks: 24)**

General Instructions:

1. Please check that this Question Paper contains 55 Questions.
2. Question Paper Code given above should be written on the Answer Book, in the space provided, by the Candidate.
3. For candidates without an Internal Assessment, their marks will be multiplied by 1.25 to adjust their total to a maximum of 100 marks.
4. 15 minutes time is given for the candidates to read the Question paper. The Question Paper will be distributed 15 minutes before the scheduled time of the examination. In these 15 minutes, the candidates should only read the instructions and questions carefully and should not write answers on the Answer Sheet.
5. The Question Paper contains 4 sections, Section A, B, C and D.
6. Section-A contains Multiple Choice Questions (MCQ). Choose the most appropriate answer from the given options. The answers to this Section must be provided in the boxes provided in the Answer Sheet. Answers provided anywhere else will not be counted for marking.
7. Section-B contains Very Short Answer Questions. Answer the questions briefly, in minimum 3 steps.
8. Section-C contains Short Answer Questions. Answer the questions in minimum 5 steps.
9. Section-D contains Long Answer Questions. Answer the questions in minimum 8 steps.
10. Use of calculators/ mobile phone/ any electronic device is NOT ALLOWED.

Section- A

Multiple Choice Questions: Attempt **ALL** Questions. (30 X 1 = 30 marks)

1. The HCF of the smallest composite number and the smallest prime number is :

   (A) 1 (B) 2 (C) 4 (D) 8

2. Which of the following is a polynomial?

   (A) $x + 7$ (B) $x^2 - 2\sqrt{x} - 1$ (C) $x + 1/x$ (D) $x^{-2} + 5x - 11$

3. A polynomial of degree 1 is called a:

   (a) Linear polynomial (B) quadratic polynomial (C) cubic polynomial (D) biquadratic polynomial

4. A quadratic polynomial can have at most:

   (a) 1 zero (B) 2 zeroes (C) 3 zeroes (D) 4 zeroes

5. The pair of equations $x = 0$ and $y = 0$ has:

   (a) Infinitely many solutions (B) two solutions (C) one solution (D) no solution

6. The system of equations $-3x + 4y = 5$ and $\frac{9}{2}x - 6y + \frac{15}{2} = 0$ has :

   (A) Unique solution (B) infinite many solutions (C) no solutions (D) none of these

7. The sum of the roots of the equation $x^2 - 6x + 5 = 0$ is :

   (A) 5 (B) - 5 (C) 6 (D) -6

8. The sum of a number and its reciprocal is 10/3 then the number is:

   (A) 5 (B) 2 (C) 6 (D) 3

9. If $x = 3$ is a solution of the quadratic equation $3x^2 + (k - 1)x + 9 = 0$, then k equals to:

   (A) – 1 1 (B) 11 (C) – 13 (D) 13

10. The common difference of the A P: - 5, - 1, 3, 7, ………… Will be:

    (A) 1 (B) 2 (C) 3 (D) 4

11. All geometrical congruent figures are:

    (A) Not similar (B) similar (C) unequal (D) none of the above

12. The corresponding sides of two similar triangles are in the ratio 4:9, then areas of these triangles are in the ratio:

    (A) 2:3 (B) 4:9 (C) 16:81 (D) 9:4

13. If two angles of one triangle are respectively equal to two angles of another triangle then the two Triangles are similar. This is referred to as:

    (A) AA Similarity Criterion for two triangles

(B) SAS Similarity Criterion for two triangles

(C) AAA Similarity Criterion for two triangles

(D) SSS Similarity Criterion for two triangles

14. The distance of a point P (3, 4) from origin is:

(A) 1 unit (B) 3 units (C) 5 units (D) 7 units

15. The coordinates of the midpoint of the line segment joining the points A (7, 0) and B (- 5, 4) is:

(A) (1, 2) (B) (4, 2) (C) (4, - 2) (D) (- 1, - 2)

16. The value of $1 + \tan^2 45^0$ is:

(A) 0 (B) – 1 (C) 1 (D) 2

17. If $\cos \theta = 1$ then the value of θ is:

(A) 0^0 (B) 30^0 (C) 60^0 (D) 90^0

18. How many tangents can be drawn parallel to the secant of a circle?

(A) One (B) two (C) three (D) infinitely many

19. If a tangent PQ at a point P to a circle with Centre O cuts a line through O at Q such that PQ = 24 cm and OQ = 25 cm then the radius of a circle is:

(A) 12 cm (B) 10 cm (C) 7 cm (D) 5 cm

20. If two tangents inclined at an angle of 60^0 are drawn to a circle of radius 3 cm, then the length of each tangent is equal to:

(A) $\frac{1}{2}\sqrt{3}$ cm (B) 3 cm (C) 6 cm (D) $3\sqrt{3}$ cm

21. A garden roller has circumference of 4 m. the number of revolutions t makes in 40 meters is:

(A) 16 (B) 12 (C) 10 (D) 8

22. If the circumference of a circle increases from 2π to 4π then its area is:

(A) four times (B) tripled (C) doubled (D) halved

23. The outer surface area of a spherical shell of radius R units is:

(A) πR^2 sq. units (B) $2\pi R^2$ sq. units (C) $3\pi R^2$ sq. units (D) $4\pi R^2$ sq. units

24. During conversion of a solid from one shape to another, the volume of the new shape will:

(A) Increase (B) decreases (C) remain unaltered (D) be doubled

25. If the surface area of a sphere is 616 cm^2 its diameter is:

(A) 7 cm (B) 14 cm (C) 28 cm (D) 56 cm

26. The middle most observation of every data arranged in order is called:

(A) Median (B) mode (C) mean (D) deviation

27. A numerical data is sad to be bimodal if it has:

(A) Single mode (B) two modes (C) three modes (D) more than three modes

28. Which of the following cannot be the probability of an event?

(A) 2/3 (B) – 1.5 (C) 15 % (D) 0.7

29. A jar contains 6 red, 5 black and 3 green marbles of equal size. The probability that a randomly drawn marble would be green in colour is:

(A) 5/14 (B) 11/14 (C) 3/14 (D) 6/14

30. The probability of an impossible event is:

(A) 0 (B) 1/2 (C) 1 (D) non- existent

Section – B

Very Short Answer Questions: Answer **any 6 (six)**. (2x6=12 marks)

31. Solve the following system of linear equations:

$2x + y = 7$ and $4x - 3y + 1 = 0$

32. Solve by factorisation: $6x^2 - x - 2 = 0$.
33. The product of two consecutive positive integers is 240. Formulate the quadratic equation whose roots are these integers.
34. Prove that $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$
35. If $\sin\theta = 3/5$, find the value of $\frac{\sin\theta\ \cos\theta)}{2\ \cot\theta}$
36. A ΔABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ΔABC is a right angled triangle?
37. A die s thrown once. What is the probability of getting: (i) an even number and (ii) a number less than 5
38. given that HCF (306, 657) = 9, find LCM of 306 and 657?
39. Express 16380 as product of prime numbers.

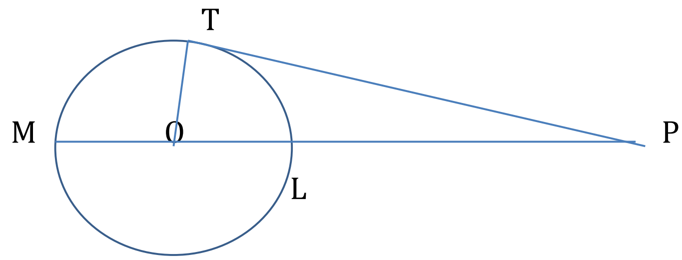
Section – C

Short Answer Questions: Answer **any 6 (six)**. **(**3x6=18 marks)

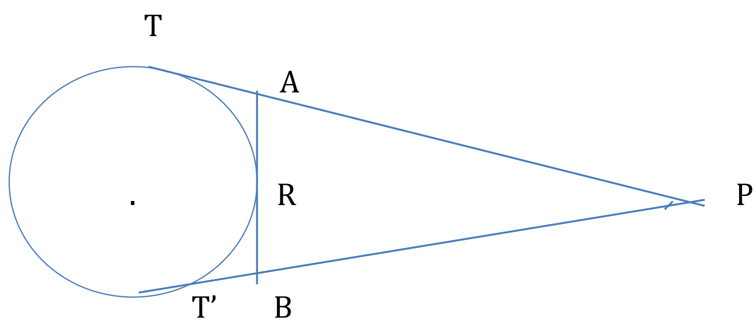
40. Find the ratio in which the point P (- 6, a) divide the join of A (- 3, - 1) and B (- 8, 9). Also find the value of a.
41. If the point P (x, y) is equidistant from the points A(5, 1) and B (1, 5), prove that x = y.

42. Mary is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages?

43. In the adjoining figure, O is the canter of the circle, PT s the tangent and PLM is the secant passing through Centre O. If PT = 8cm and PL = 4cm, then find the radius of the circle?



44. In the adjoining figure, PT and PT' are tangents from P to the circle with Centre O. R are a point on the circle where AB is another tangent. Prove that PA + AR = PB + BR.



45. Find the area of the sector of a circle with radius 4 cm and of angle 30^0. Also, find the area of the corresponding major sector? (use π = 22/7)

46. The following table shows the ages of the patients admitted in a hospital during the year:

Ages (in years)	Number of patients
5-15	6
15-25	11
25-35	21
35-45	23
45-55	14
55-65	5

Based on the above information, answer the following:

(i) Write the modal class. (1)

(ii) Find mode of the given data. (2)

47. If α and β are zeroes of the polynomial $P(x) = 3x^2 - 2x - 6$, then find the value of $1/\alpha + 1/\beta$.

48. Examine whether the quadratic equation $\mathbf{2x^2 + x - 6 = 0}$ have real roots. If so, find the roots.

Section – D

Long Answer Questions: Answer **any 4 (four)** (4x5=20 marks)

49. A toy is in the form of a hemisphere surmounted by a right circular cone of the same base radius as that of the hemisphere. If the radius of the base of a cone is 21 cm and its volume is 2/3 of the volume of the hemisphere, calculate the height of the cone and surface area of the toy. (Use $\pi = 22/7$).

50. The fourth term of an AP is equal to 3 times the first term and seventh term exceeds twice the third tem by 1. Find the first term and the common difference. Also, find the sum of the first 10 terms.

51. The 24^{th} term of an AP is twice its 10^{th} term. Show that its 72^{nd} term is 4 times its 15^{th} term.

52. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

53. In the given figure PA, QB ad RC are each perpendicular to AC, such that PA =x, QB = z, RC = y, AB= a, and BC = b. prove that $1/x + 1/y = 1/z$.

54. The angle of elevation of the top of a tower from a point on the ground is 30^0. On moving a distance of 20 metres towards the foot of the tower to a point B the angle of elevation increases to 60^0. Find the height of the tower and the distance of the tower from the point A. (use $\sqrt{3} = 1.732$)

55. State and prove basic proportionality theorem.

* End of Question Paper *